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FRACTIONS OF ARITHMETIC OCTAHEDRON AND RANDOM WALK

Alexander Yurkin*
City of Puschino, Russia
alvl1yurkin@rambler.ru

ABSTRACT
The linear and nonlinear 3D random walk in completely filled with numbers arithmetic octahedrons composed of small cubes is considered. It is shown that the numbers in a linear arithmetic octahedron are two independent fractions. It is shown that the numbers in the nonlinear arithmetic octahedron also comprise two independent fractions which in turn consist of seven sub-fractions. The first fraction consists of three, and the second of four independent sub-fractions. However in subsequent iterations the numbers constituting the sub-fractions can be added together within their fraction. The possibility of fractions and sub-fractions of numbers in 1D and 2D random walk is discussed.

KEYWORDS:
deterministic model, arithmetic octahedrons, geometrization of physics, random walk, nonlinear effects, fractions of numbers

1. INTRODUCTION

The regular octahedron [1] refers to the number of five Platonic figures. It can be composed of eight equal equilateral triangles or twelve identical segments. "The octahedron is dual to the cube" [1]. The regular octahedron can also be composed of many identical small cubes just as in Ancient Egypt were pyramids of stone blocks. The construction of an octahedron using small cubes can be obtained by considering a random walk in three-dimensional (3D) space. In [2] we considered a visual model of a 3D random linear and nonlinear walk in an octahedron. In [3] we reviewed and systematized the visual models of 1D, 2D, and 3D random linear and nonlinear walks.

In this paper we explore some new features, patterns, and fractions of numbers in visual 3D models of random linear and nonlinear walks in an octahedron composed of small cubes.

2. FRACTIONS OF NUMBERS IN A LINEAR RANDOM WALK

In papers [2, 3] linear random 3D walk (forward, backward, right, left, up, down) was considered in a linear octahedron composed of small identical cubes.

In [2, 3] we considered only one fraction of numbers; therefore our arithmetic linear octahedron was not completely filled with cubes and contained gaps only in neighboring cells in its volume. In this paper we consider two fractions of numbers in the volume of an octahedron with the help of which the volume of an octahedron is filled completely.

A 3D linear random walk or a walk in a volume can be described using a 3D. We obtain the linear binomial coefficients in the computed cell of the octahedron n by summing the numbers from the six cells adjacent to the computed cell.

2.1. 1-FRACTION IN A LINEAR RANDOM WALK

Figures 1 - 4 show sequentially (for the first four iterations) images of linear arithmetic octahedrons composed of small cubes. (a) shows images of the arithmetic octahedrons themselves, (b) shows images of layers of

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octahedrons composed of small cubes containing numbers. These numbers correspond to the number of walks from the initial cell (initial cube) to the final cell (final cube).

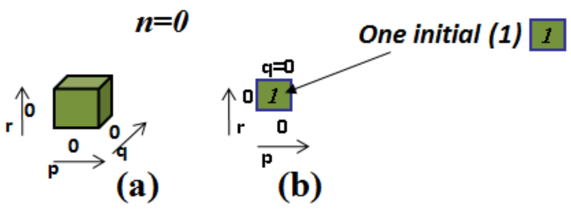

Figure 1. The zero linear arithmetic octahedron (the zero iteration $n = 0$). 1-fraction.

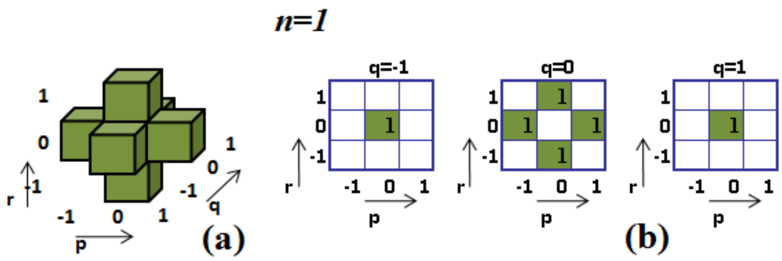

Figure 2. The first linear arithmetic octahedron (the first iteration $n = 1$). 1-fraction

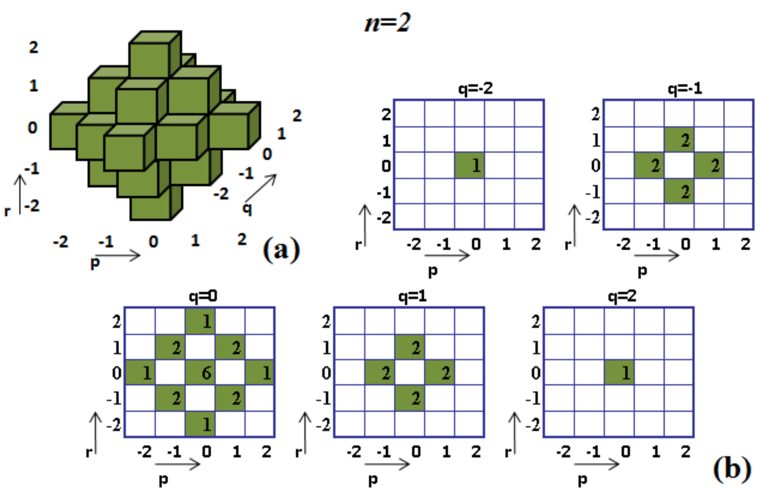

Figure 3. The second linear arithmetic octahedron (the second iteration $n = 2$). 1-fraction.

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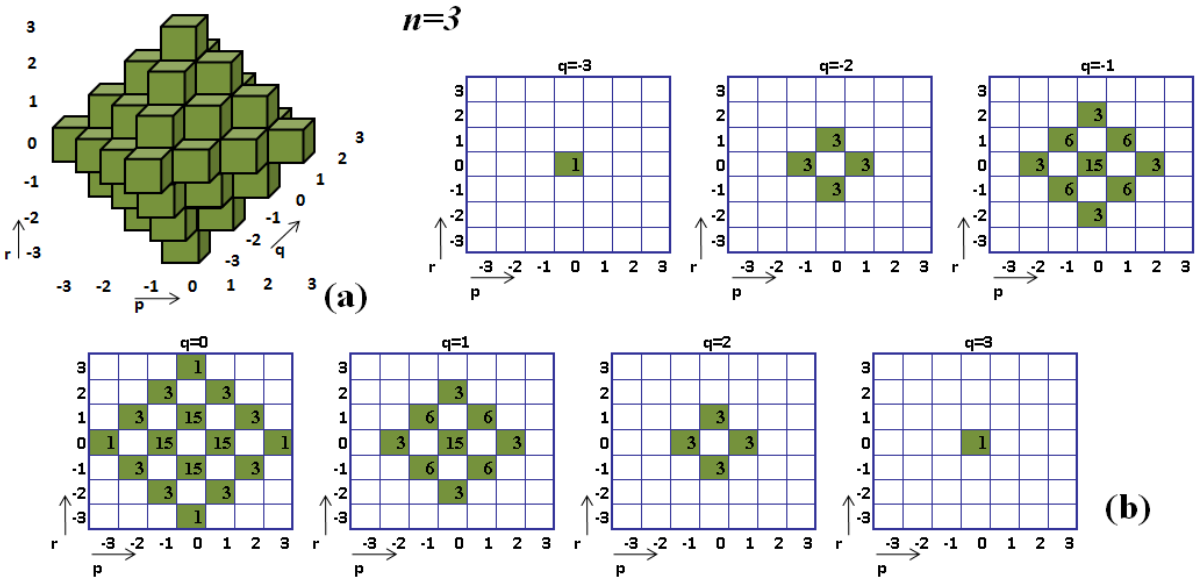

Figure 4. The third linear arithmetic octahedron (the third iteration $n=3$). 1-fraction.

The sequence of numbers of octahedrons for the 3D case in this example is denoted by $n := 0, 1, 2, \dots$. The numbers characterizing the octahedron (the numbers characterizing the position of the cubes of which the octahedron is composed) are denoted p, q and r:

$$p = 0, \pm1, \pm2, \dots, \pm\{n - |q|\};\ q = 0, \pm1, \pm2, \dots, \pm n;\ r = 0, \pm1, \pm2, \dots, \pm\{n - |q|\}. \quad (1)$$

Denote a number located in the n – octahedron as $\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix}$.

The numbers in the linear arithmetic octahedron are linear binomial coefficients $\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix}$ can be found [2, 3] using the recursive 3D linear expression:

$$\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix} = \begin{pmatrix} n-1 \\ p \\ q-1 \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q \\ r+1 \end{pmatrix} + \begin{pmatrix} n-1 \\ p-1 \\ q \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p+1 \\ q \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q \\ r-1 \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q+1 \\ r \end{pmatrix}. \quad (2)$$

Then we specify the first fraction of the numbers of the zero octahedron ($n = 0$) or in other words the initial conditions:

$$\begin{pmatrix} 0 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for } p = 0, q = 0, r = 0 \text{ and } \begin{pmatrix} 0 \\ p \\ q \\ r \end{pmatrix} = 0 \quad (3)$$

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for the other values p, q and r. These initial conditions are shown in Figure 1. They provide the appearance of a group of numbers shown in Figures 1 - 4 after performing recurrent calculations. Assign the name “1-fraction” to this group of numbers. 1-fraction cubes in Figures 1 - 4 are shown in green.

The octahedrons depicted in Figures 1 - 4 contain regular gaps in adjacent cells (cubes without numbers) since the octahedrons consist of only one fraction (1-fraction) of numbers.

2.2. 2-FRACTION IN A LINEAR RANDOM WALK

Figures 5 - 8 shows the construction of an arithmetic linear octahedron similar to the construction of an octahedron on Figures 1 - 4 but consisting of two fractions (groups) of numbers.

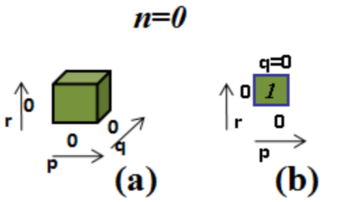

Figure 5. The zero linear arithmetic octahedron (the zero iteration $n = 0$). 1-fraction.

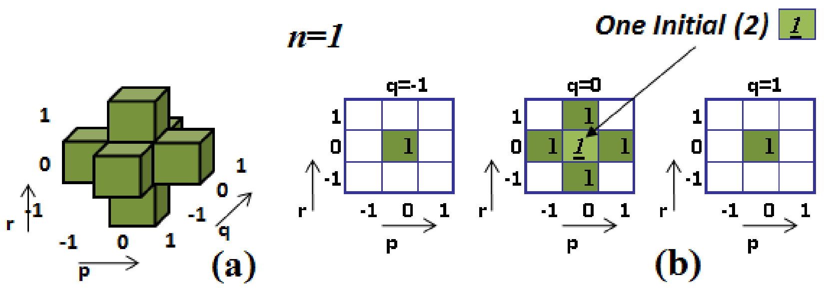

Figure 6. The first linear arithmetic octahedron (the first iteration $n = 1$). 1-fraction on the surface of octahedron and 2-fraction inside it.

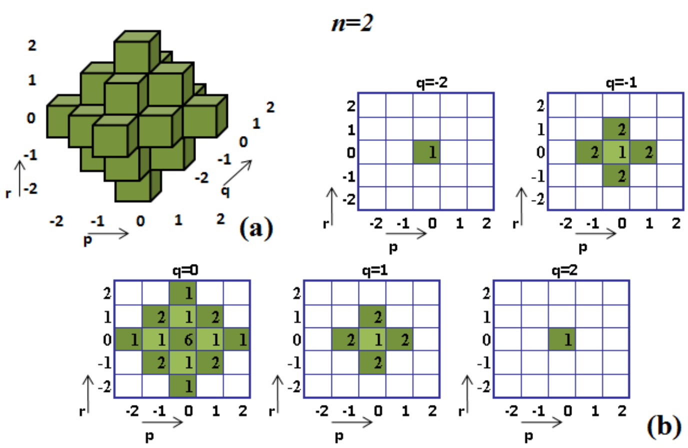

Figure 7. The second linear arithmetic octahedron (the second iteration $n = 2$). 1-fraction on the surface of octahedron and 2-fraction inside it.

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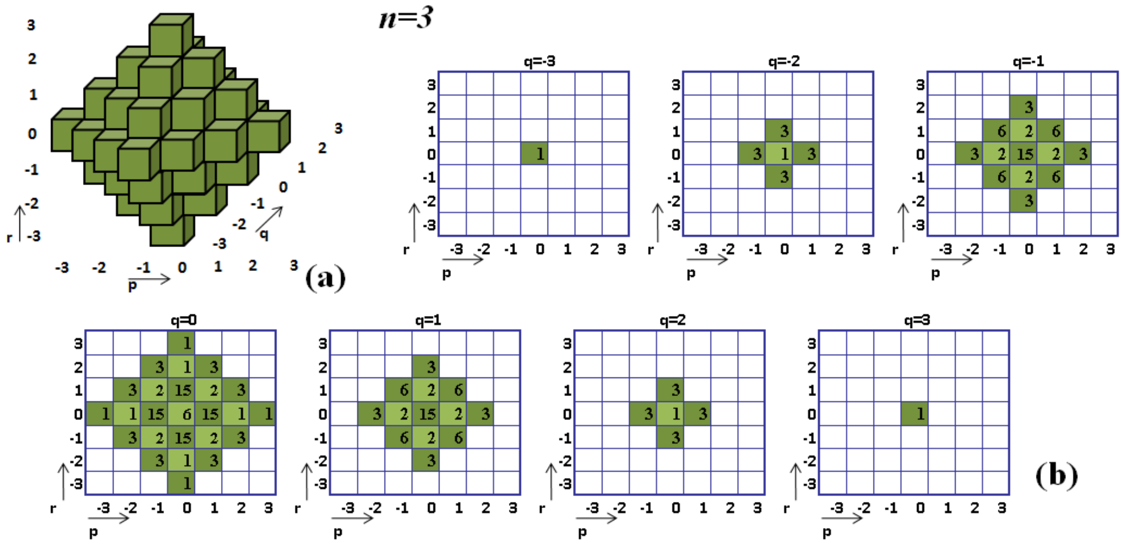

Figure 8. The third linear arithmetic octahedron (the third iteration $n = 3$). 1-fraction on the surface of octahedron and 2-fraction inside it.

To construct this linear octahedron we use the same notation (1), recurrent expression (2), and initial conditions (3) as in the previous case.

We specify the additional number of the first octahedron ($n = 1$) or in other words the additional initial conditions of the second fraction of the numbers:

$$\begin{pmatrix} 1 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for } p = 0, q = 0, r = 0. \qquad (4)$$

These initial conditions are shown in Figure 6. These second initial conditions ensure the appearance of the second group of numbers shown in Figures 6 - 8 after the implementation of recurrent calculations. Assign this group of numbers the name “2-fraction. The 2-fraction cubes on Figures 6 - 8 are shown in light green.

The octahedrons depicted in Figures 6 - 8 do not contain regular gaps (cubes without numbers) since they consist of two fractions (1-fraction and 2-fraction) numbers. Volumes of octahedrons are full by numbers.

The numbers 1-fraction and 2-fraction of the linear octahedron are independent of each other and do not affect each other. The numbers of these two different fractions are not summarized during subsequent iterations. All numbers obtained using expressions (1 - 4) do not go beyond the octahedrons.

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3. FRACTIONS OF NUMBERS IN A NONLINEAR RANDOM WALK

In papers [2, 3] a nonlinear random 3D walk (forward, backward, right, left, up, down) was considered in a nonlinear octahedron composed of small identical cubes.

In [2, 3] we considered only one fraction of numbers; therefore our arithmetic nonlinear octahedron was not completely filled with cubes and contained gaps not only in neighboring cells but also in other cells in its volume. In this paper we consider two fractions of numbers in the volume of an octahedron with the help of which the volume of an octahedron is filled completely by numbers. However these two fractions consist of sub-fractions; the first fraction consists of three and the second of four sub-fractions.

A 3D nonlinear random walk or a walk in a volume can also be described like a linear one using a 3D model in the form of a nonlinear arithmetic regular octahedron [2, 3]. Nonlinear binomial coefficients in the computed cell of the octahedron $n = 1$ are obtained by summing the numbers of six cells adjacent to the computed cell; nonlinear binomial coefficients in the cell of the octahedron $n = 2$ we get by summing the numbers of six cells located one through the calculated cell; nonlinear binomial coefficients in the cell of the octahedron $n = 3$ we get by summing the numbers of six cells located two through the calculated cell; etc.

3.1. 1-FRACTION IN A NONLINEAR RANDOM WALK

Figures 9 - 12 show sequentially (for the first four iterations) images of the nonlinear arithmetic octahedrons composed of small cubes. (a) shows images of the arithmetic octahedrons themselves, (b) shows images of layers of octahedrons composed of cubes containing numbers. These numbers correspond to the number of walks from the initial cell (initial cube) to the final cell (final cube).

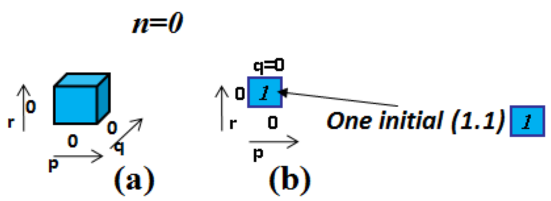

Figure 9. The zero nonlinear arithmetic octahedron (the zero iteration $n = 0$). 1.1-sub-fraction of 1-fracton.

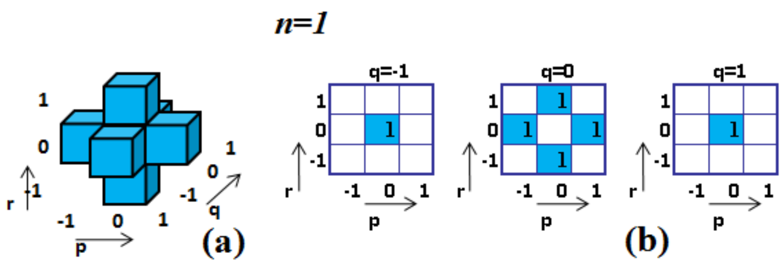

Figure 10. The first nonlinear arithmetic octahedron (the first iteration $n = 1$). 1.1-sub-fraction of 1-fracton.

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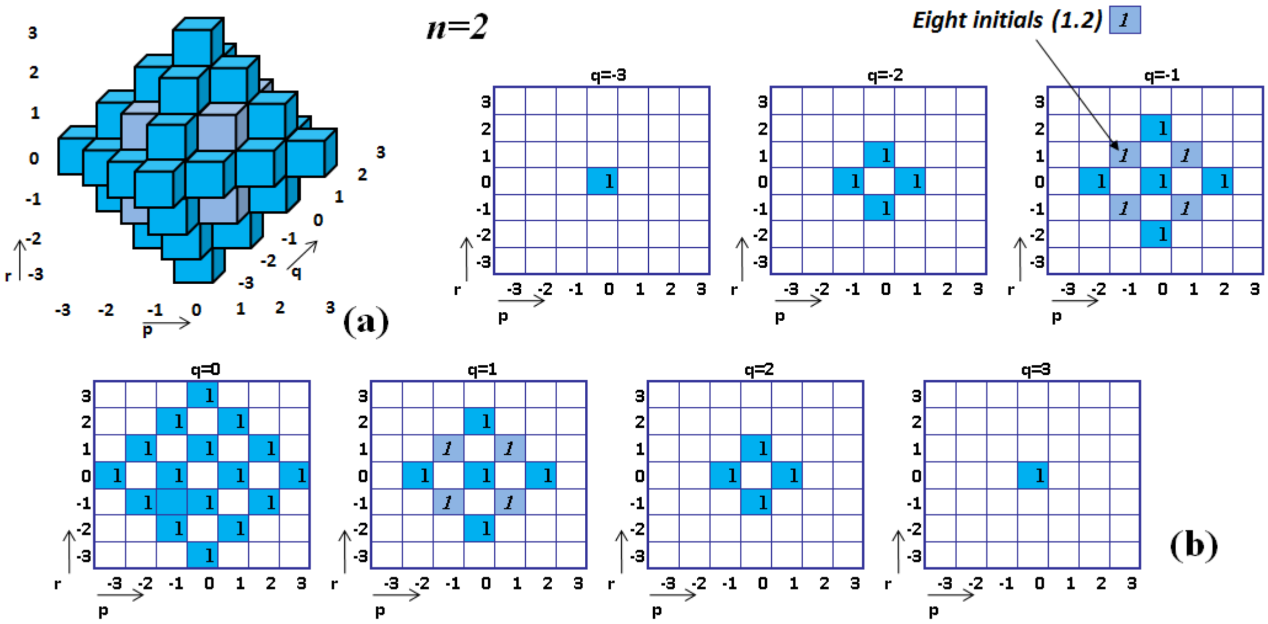

Figure 11. The second nonlinear arithmetic octahedron (the second iteration $n=2$). 1.1-sub-fraction and 1.2-sub-fraction of 1-fracton.

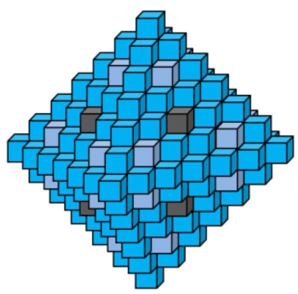

Figure 12a. The image of the third nonlinear arithmetic octahedron (the third iteration $n=3$). 1.1-sub-fraction, 1.2-sub-fraction and 1.3-sub-fraction of 1-fracton.

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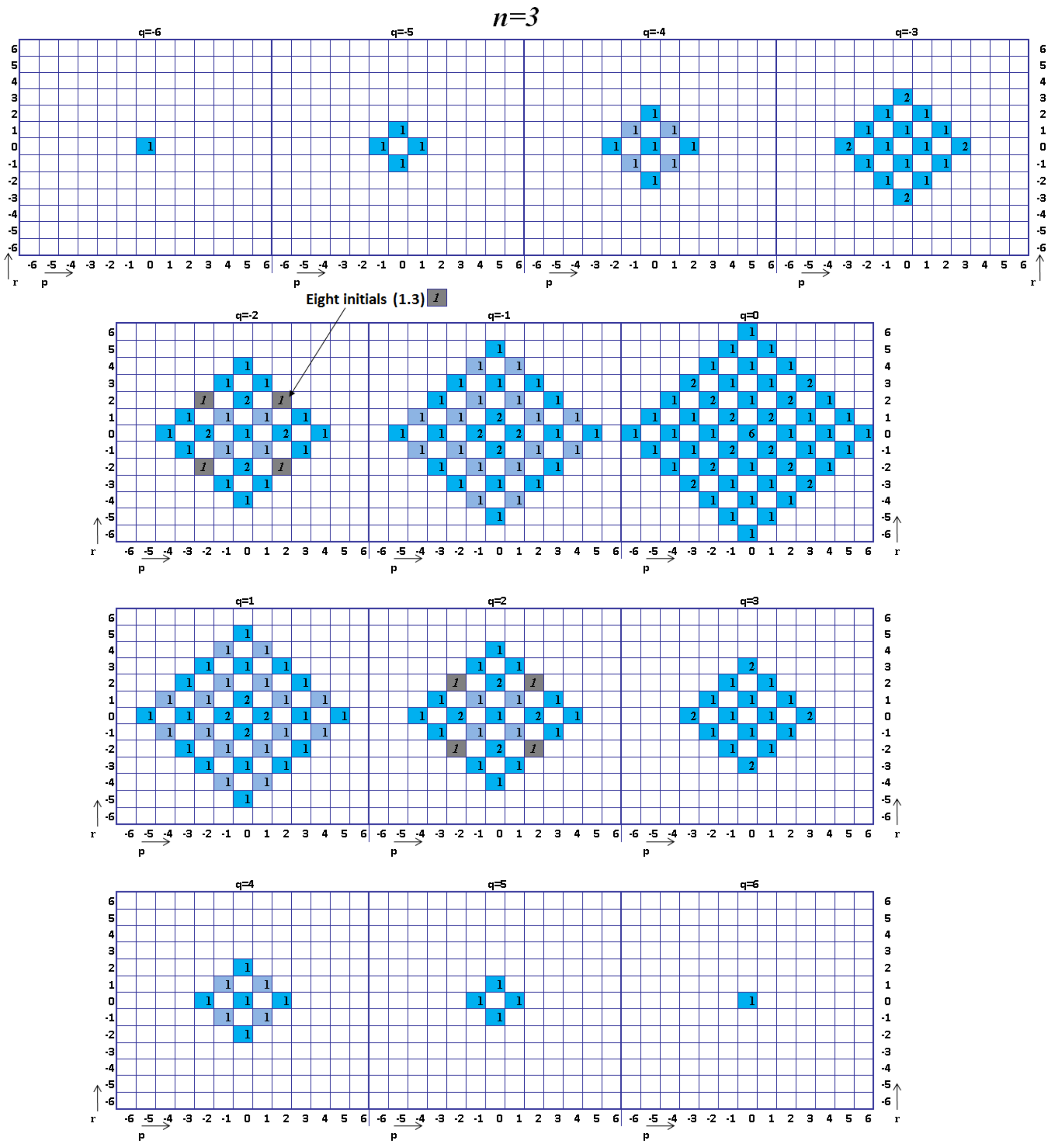

Figure 12b. The images of layers of the third nonlinear arithmetic octahedron (the third iteration $n=3$). 1.1-sub-fraction, 1.2-sub-fraction and 1.3-sub-fraction of 1-fracton.

The sequence of numbers of octahedrons (rows of numbers in the octahedron) for the 3D case in this example is denoted by n: $n = 0, 1, 2, \ldots$. The numbers characterizing the nonlinear octahedron (the numbers characterizing the position of the cubes of which the octahedron is composed) are denoted by p, q and r:

$$\left.\begin{array}{l} p = 0, \pm 1, \pm 2, \ldots, \pm\{\left[\frac{n(n+1)}{2}\right] - |\, q\, |\}; \\ q = 0, \pm 1, \pm 2, \ldots, \pm n(n+1)/2; \end{array}\right\} \quad (5)$$

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$$r = 0, \pm 1, \pm 2, \ldots, \pm\{\left[\frac{n(n+1)}{2}\right] - |\ q\ |\}.$$

Denote a number located in the n - octahedron as $\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix}$.

The numbers in the nonlinear arithmetic octahedron are nonlinear binomial coefficients $\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix}$ can be found [2, 3] using the recursive 3D nonlinear expression:

$$\begin{pmatrix} n \\ p \\ q \\ r \end{pmatrix} = \begin{pmatrix} n-1 \\ p \\ q-n \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q \\ r+n \end{pmatrix} + \begin{pmatrix} n-1 \\ p-n \\ q \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p+n \\ q \\ r \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q \\ r-n \end{pmatrix} + \begin{pmatrix} n-1 \\ p \\ q+n \\ r \end{pmatrix}. \quad (6)$$

Then we specify the number of the zero octahedron ($n = 0$) or in other words the initial conditions of the first sub-fraction of the first fraction of numbers:

$$\begin{pmatrix} 0 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for } p = 0, q = 0, r = 0 \text{ and } \begin{pmatrix} 0 \\ p \\ q \\ r \end{pmatrix} = 0 \quad (7)$$

for the other values of p, q and r. These initial conditions are shown in Figure 9. They provide the appearance of a group of numbers shown in Figures 9 - 12 after performing recurrent calculations. Assign the name “1.1-sub-fraction” to this group of numbers. The 1.1-sub-fraction cubes in Figures
9 -12 are shown in blue.

Then we specify the additional numbers of the second octahedron ($n = 2$) or in other words the additional initial conditions of the second sub-fraction of the first fraction of numbers:

$$\begin{pmatrix} 2 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for} \quad (8)$$

p=	-1	-1	-1	-1	1	1	1	1
q=	-1	-1	1	1	-1	-1	1	1
r=	-1	1	-1	1	-1	1	-1	1

These eight initial conditions are shown in Figure 11. They provide the appearance of the second group of numbers depicted in Figures 11, 12 after the implementation of recurrent calculations. Assign the name “1.2-sub-fraction” to this group of numbers. The 1.2-sub-fraction cubes in Figures
11, 12 are shown in light blue.

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Then we specify the additional numbers of the third octahedron ($n = 3$) or in other words the additional initial conditions of the third sub-fraction of the first fraction of numbers:

$$\begin{pmatrix} 3 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for} \tag{9}$$

p=	-2	-2	-2	-2	2	2	2	2
q=	-2	-2	2	2	-2	-2	2	2
r=	-2	2	-2	2	-2	2	-2	2

These eight initial conditions are shown in Figure 12. They provide the appearance of the third group of numbers, shown in Figure 12. Give this group of numbers the name “1.3-sub-fraction”. The 1.3-sub-fraction cubes in Figure 12 are shown in gray.

The octahedrons depicted in Figures 10 - 12 contain gaps (cubes without numbers) only in neighboring cells as in the linear case (paragraph 2.1) since they consist of three sub-fractions of the first fraction (1.1-sub-fraction, 1.2-sub-fraction and 1.3-sub-fraction) numbers.

Our numerical calculations for large values of n have shown: The numbers of three sub-fractions do not depend on each other and do not influence each other. All cells of the octahedrons except neighboring ones are completely filled if the initial conditions for 1.1-sub-fraction, 1.2-sub-fraction and 1.3-sub-fraction are specified.

3.2. 2-FRACTION IN A NONLINEAR RANDOM WALK

Figures 13 - 17 show the construction (for the first five iterations) of an arithmetic nonlinear octahedron similar to the construction of an octahedron in Figures 9 - 12 but consisting of two fractions (groups) of numbers.

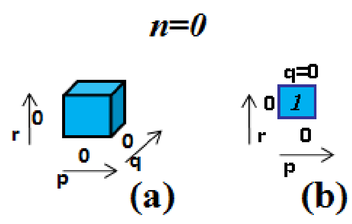

Figure 13. The zero nonlinear arithmetic octahedron (the zero iteration $n = 0$). 1.1-sub-fraction of 1-fracton.

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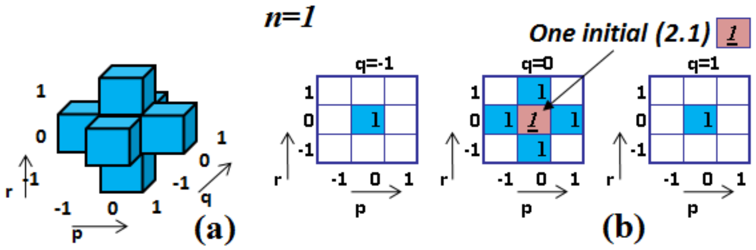

Figure 14. The first nonlinear arithmetic octahedron (the first iteration $n = 1$). 1.1-sub-fraction of 1-fracton and 2.1-sub-fraction of 2-fracton.

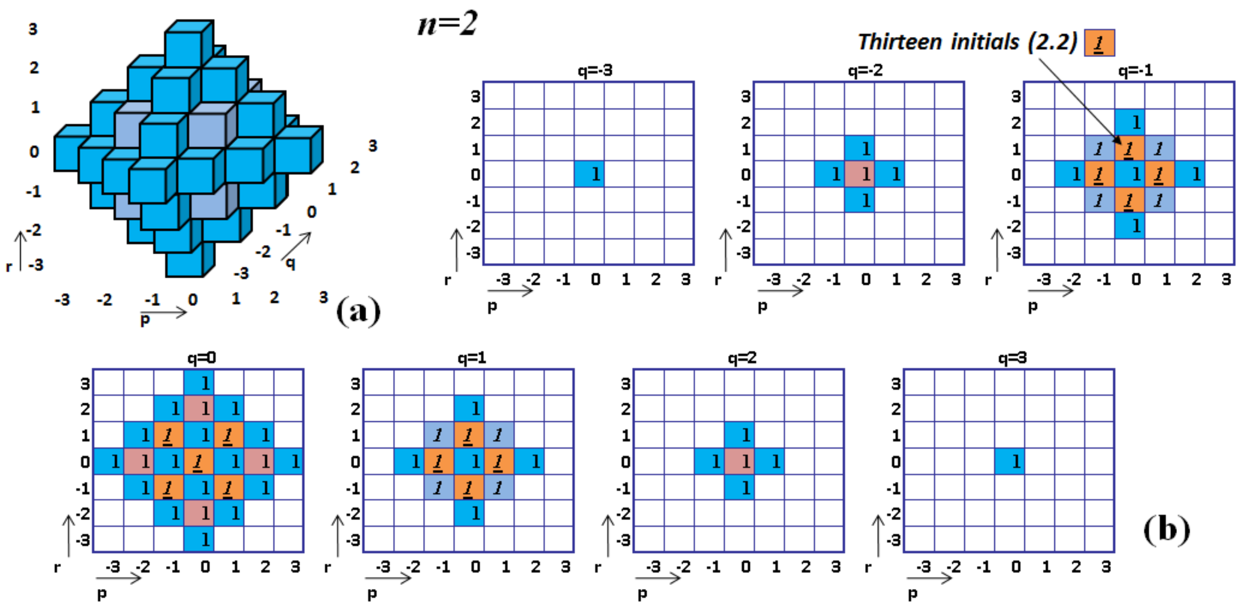

Figure 15. The second nonlinear arithmetic octahedron (the second iteration $n = 2$). 1.1-sub-fraction, 1.2-sub-fraction of 1-fracton, and 2.1-sub-fraction, 2.2-sub-fraction of 2-fracton.

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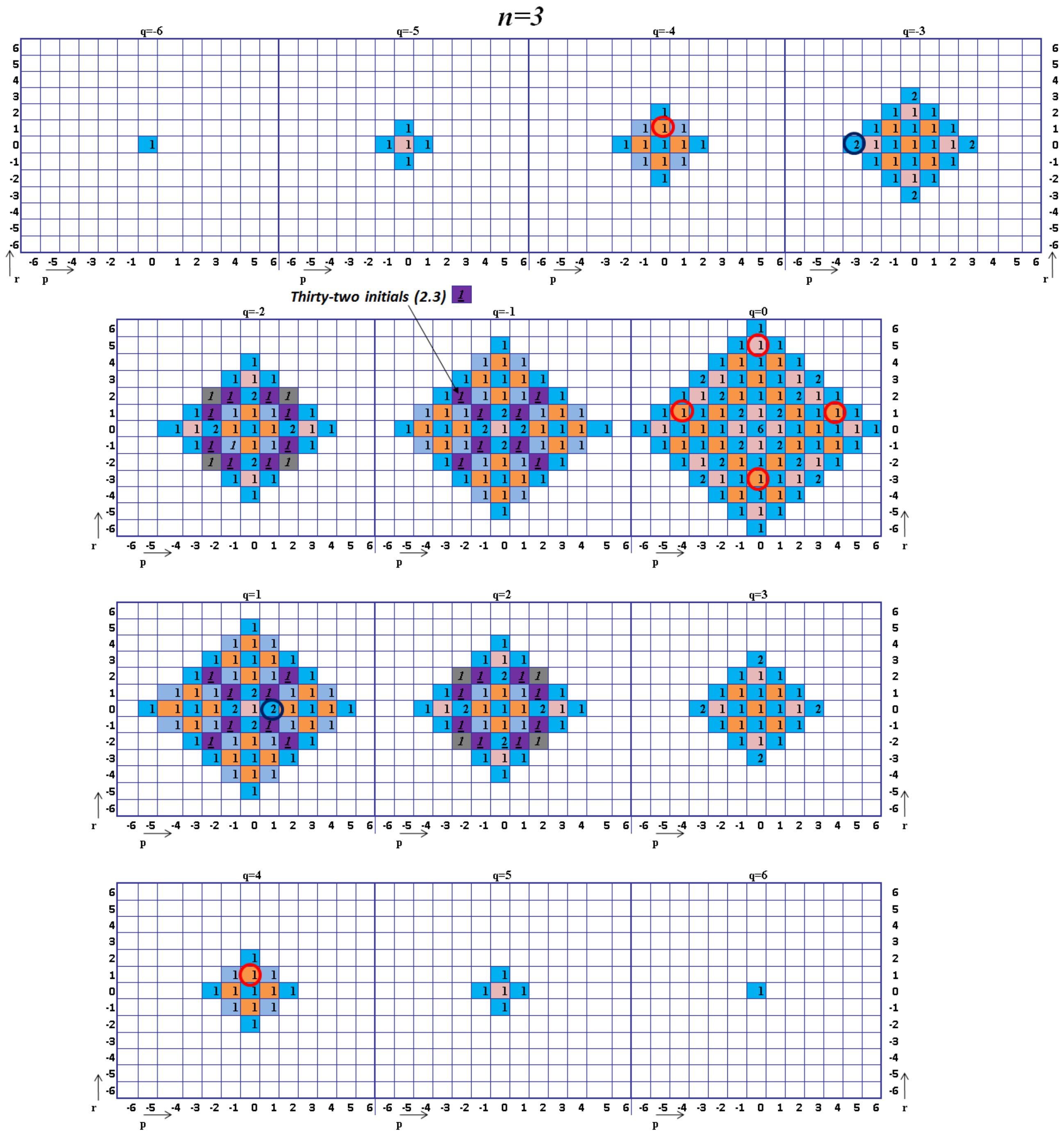

Figure 16. The images of layers of the third nonlinear arithmetic octahedron (the third iteration $n = 3$). 1.1-sub-fraction, 1.2-sub-fraction, 1.3-sub-fraction of 1-fracton, and 2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction of 2-fracton. The image of the third nonlinear arithmetic octahedron is the same as in Figure 12a.

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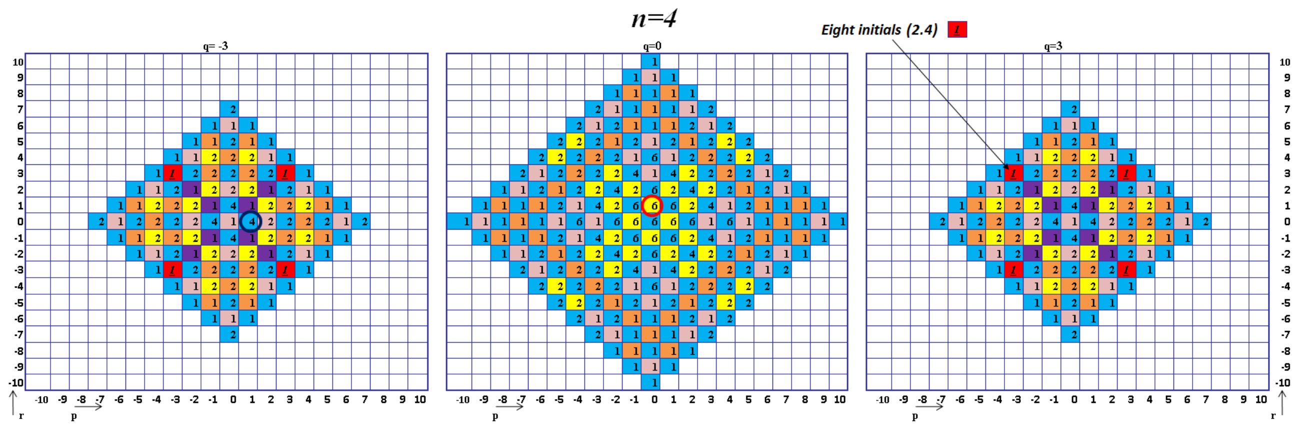

Figure 17. The images of layers of the fourth nonlinear arithmetic octahedron (the fourth iteration $n = 4$). 1.1-sub-fraction of 1-fracton, and 2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction, 2.4-sub-fraction of 2-fracton. To save space, only three out of 21 layers are shown; the layers $q = \pm 3$ contain the initial conditions for 2.4-sub-fraction. Yellow light highlights: 1) cells where numbers of 1.1-sub-fraction and 1.2-sub-fraction are summed in the layers $q = \pm 3$; 2) cells where numbers of 2.1-sub-fraction and 2.2-sub-fraction are summed in the layer $q = 0$.

To construct this nonlinear octahedron we use the same notation (5), recurrent expression (6), and initial conditions (7 - 9) as in the previous case.

However we set additional initial conditions for the second fraction. We specify the additional number of the first octahedron ($n = 1$) or in other words the additional initial conditions for the second fraction of the numbers:

$$\begin{pmatrix} 1 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for } p = 0, q = 0, r = 0. \tag{10}$$

These initial conditions are shown in Figure 14. These additional initial conditions ensure the appearance of the first sub-fraction of the second fraction of the numbers shown in Figures 14 - 17 after the implementation of recurrent calculations. Assign this group of numbers the name "2.1-sub-fraction. The 2.1-sub-fraction cubes on Figures 14 - 17 are shown in pink.

Then we specify the additional numbers of the second octahedron ($n = 2$) or in other words the additional initial conditions of the second sub-fraction of the second fraction of numbers:

$$\begin{pmatrix} 2 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for:} \tag{11}$$

p=	-1	0	0	1	1	-1	0	1	-1	-1	0	0	1
q=	-1	-1	-1	-1	0	0	0	0	0	1	1	1	1
r=	0	1	-1	0	1	-1	0	1	-1	0	1	-1	0

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These thirteen initial conditions are shown in Figure 15. They belong to the second sub-fraction of the second fraction of numbers depicted in Figures 15 - 17. Assign the name “2.2-sub-fraction” to this group of numbers. The 2.2-sub-fraction cubes on Figures 15–17 are shown in light brown.

Then we specify the additional numbers of the third octahedron ($n = 3$) or in other words the additional initial conditions of the third sub-fraction of the second fraction of numbers:

$$\begin{pmatrix} 3 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for:} \tag{12}$$

p=	-2	-2	-1	-1	1	1	2	2	-2	-2	-1	-1	1	1	2	2
q=	-2	-2	-2	-2	-2	-2	-2	-2	-1	-1	-1	-1	-1	-1	-1	-1
r=	1	-1	2	-2	2	-2	1	-1	2	-2	1	-1	1	-1	2	-2
p=	-2	-2	-1	-1	1	1	2	2	-2	-2	-1	-1	1	1	2	2
q=	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
r=	2	-2	1	-1	1	-1	2	-2	1	-1	2	-2	2	-2	1	-1

These thirty-two initial conditions are shown in Figure 16. They belong to the third sub-fraction of the second fraction of numbers shown in Figure 16, 17. Assign the name "2.3-sub-fraction" to this group of numbers. The 2.3-sub-fraction cubes in Figure 16, 17 are shown in purple.

Then we specify the additional numbers of the fourth octahedron ($n = 4$) or in other words the additional initial conditions of the fourth sub-fraction of the second fraction of numbers:

$$\begin{pmatrix} 4 \\ p \\ q \\ r \end{pmatrix} = 1 \text{ for:} \tag{13}$$

p=	-3	-3	-3	-3	3	3	3	3
q=	-3	-3	3	3	-3	-3	3	3
r=	-3	3	-3	3	-3	3	-3	3

These eight initial conditions are shown in Figure 17. They belong to the fourth sub-fraction of the second fraction of numbers, shown in Figure 17. Give this group of numbers the name "2.4-sub-fraction". The 2.4-sub-fraction cubes in Figure 17 are shown in red.

The octahedrons depicted in Figures 14 - 17 do not contain gaps (cubes without numbers) as in the linear case (paragraph 2.2) since they consist of two fractions (1-fraction and 2-fraction) of numbers. 1-fraction and 2-fraction consist in turn of seven sub-fractions (1.1-sub-fraction, 1.2-sub-fraction, 1.3-sub-fraction and 2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction, 2.4-sub-fraction) of numbers. Volumes of octahedrons are filled with numbers completely.

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Example 1: $n = 4, p = 0, q = 0, r = 1$. In accordance with the expression (6) we have:

$$\begin{pmatrix}4\\0\\0\\1\end{pmatrix} = \begin{pmatrix}3\\0\\-4\\1\end{pmatrix} + \begin{pmatrix}3\\0\\0\\5\end{pmatrix} + \begin{pmatrix}3\\-4\\0\\1\end{pmatrix} + \begin{pmatrix}3\\4\\0\\1\end{pmatrix} + \begin{pmatrix}3\\0\\0\\-3\end{pmatrix} + \begin{pmatrix}3\\0\\4\\1\end{pmatrix} = 6,$$

as

$$\begin{pmatrix}3\\0\\-4\\1\end{pmatrix} = 1, \begin{pmatrix}3\\0\\0\\5\end{pmatrix} = 1, \begin{pmatrix}3\\-4\\0\\1\end{pmatrix} = 1, \begin{pmatrix}3\\4\\0\\1\end{pmatrix} = 1, \begin{pmatrix}3\\0\\0\\-3\end{pmatrix} = 1, \begin{pmatrix}3\\0\\4\\1\end{pmatrix} = 1.$$

In Figures 16 and 17 these numbers are circled in red circles.

Example 2: $n = 4, p = 1, q = -3, r = 0$. In accordance with the expression (6) we have:

$$\begin{pmatrix}4\\1\\-3\\0\end{pmatrix} = \begin{pmatrix}3\\1\\-7\\0\end{pmatrix} + \begin{pmatrix}3\\1\\-3\\4\end{pmatrix} + \begin{pmatrix}3\\-3\\-3\\0\end{pmatrix} + \begin{pmatrix}3\\5\\-3\\0\end{pmatrix} + \begin{pmatrix}3\\1\\-3\\-4\end{pmatrix} + \begin{pmatrix}3\\1\\1\\0\end{pmatrix} = 4,$$

as

$$\begin{pmatrix}3\\1\\-7\\0\end{pmatrix} = 0, \begin{pmatrix}3\\1\\-3\\4\end{pmatrix} = 0, \begin{pmatrix}3\\-3\\-3\\0\end{pmatrix} = 2, \begin{pmatrix}3\\5\\-3\\0\end{pmatrix} = 0, \begin{pmatrix}3\\1\\-3\\-4\end{pmatrix} = 0, \begin{pmatrix}3\\1\\1\\0\end{pmatrix} = 2.$$

In Figures 16 and 17 these numbers are circled in blue circles. Four numbers are zeroes because they are beyond the squares with numbers in Figure 16 in accordance with the expressions (5).

Our numerical calculations for large values of n showed: The numbers 1-fraction and 2-fraction of the nonlinear octahedron do not depend on each other and do not affect each other. The numbers of these two different fractions are not summarized during subsequent iterations. All numbers obtained with the help of expressions (5 - 13) do not go beyond the limits of octahedrons.

The numbers of three sub-fractions (1.1-sub-fraction, 1.2-sub-fraction, 1.3-sub-fraction) of the first fraction of the nonlinear octahedron do not depend on each other and do not affect each other. The numbers of four sub-fractions (2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction, 2.4-sub-fraction) of the second fraction of the nonlinear octahedron do not depend on each other and do not affect each other. The numbers of sub-fractions can be summed up at subsequent iterations within their fraction (Figure 17).

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CONCLUSION

Our studies of the deterministic models and visual constructions of linear (without any acceleration) and nonlinear (with the simplest uniformly acceleration) random walk and arithmetic octahedrons given in this paper show various geometric properties and nonlinear effects of 3D spaces.

1. A linear arithmetic octahedron completely filled with numbers (in a linear random walk) contains two fractions (1-fraction and 2-fraction) whose numbers are independent of each other, do not affect each other, and are not added at subsequent iterations. The initial conditions for 1-fraction and 2-fraction can be set at zero ($n = 0$) and first ($n = 1$) iterations respectively. In this case the number of initial conditions will be minimal. Our numerical calculations for large values of n have shown that if at the first ($n = 1$) iteration the linear arithmetic octahedron is completely filled with numbers then it remains completely filled with numbers even at subsequent iterations.

2. A nonlinear arithmetic octahedron completely filled with numbers (in a non-linear random walk) also contains two fractions (1-fraction and 2-fraction) the numbers of which are independent of each other, do not influence each other, and are not summed up at subsequent iterations. In turn 1-fraction contains three sub-fractions: 1.1-sub-fraction, 1.2-sub-fraction and 1.3-sub-fraction, and 2-fraction contains four sub-fractions: 2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction, and 2.4-sub-fraction.The numbers of sub-fractions do not depend on each other, do not affect each other, but can be summed up at subsequent iterations within their fraction.

The initial conditions for 1.1-sub-fraction, 1.2-sub-fraction, and 1.3-sub-fraction can be set at zero ($n = 0$), second ($n = 2$), and third ($n = 3$) iterations respectively. Initial conditions for 2.1-sub-fraction, 2.2-sub-fraction, 2.3-sub-fraction, and 2.4-sub-fraction can be set at the first ($n = 1$), second ($n = 2$), third ($n = 3$), and fourth ($n = 4$) iterations respectively. In this case the number of initial conditions will be minimal. Our numerical calculations for large values of n have shown that if at the fourth ($n = 4$) iteration the nonlinear arithmetic octahedron is completely filled with numbers then it remains completely filled with numbers even at subsequent iterations.

3. We examined the most complicated 3D case of a nonlinear random walk in this paper. In the same way it is possible to examine simpler 1D and 2D [3] cases of a nonlinear random walk. Fractions and sub-fractions of numbers exist in these 1D and 2D cases too as in 3D case.

4. Perhaps our new geometric linear and nonlinear constructions and recursive formulas will find application to understand the development of processes in biology [4] in optics [5] and acoustics [5] and also in other areas for example in technology or medicine.

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