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# Selection and Designing of Six Sigma Tightened - Normal - Tightened Variables Sampling Scheme (SSTNTVSS ( $\mathbf{n}_{\mathbf{T}}, \mathbf{n}_{\mathrm{N}} ; \mathbf{k}$ )) Indexed by Six Sigma AQL and Six Sigma AOQL 

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#### Abstract

This article gives selection and designing of Six Sigma Tightened- Normal - Tightened Variables Sampling Scheme (SSTNTVSS $\left(\mathrm{n}_{\mathrm{T}}, \mathrm{n}_{\mathrm{N}} ; k\right)$ ) indexed by Six Sigma AQL and Six Sigma AOQL. The procedure of designing and constructed tables for easy selection of system given by six sigma quality levels by known and unknown $\sigma$ at Six Sigma levels.


## Keywords

Tightened- Normal - Tightened Scheme, Variables Sampling, Six Sigma AOQ, Six Sigma AQL and Six Sigma AOQL.

## INTRODUCTION

AOQ is widely used for the evaluation of a rectifying sampling plan. The average outgoing quality is the quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective p . The maximum ordinate on the AOQ curve represents the worst possible average quality that would results from the rectifying inspection program, and this point is called the average outgoing quality limit (AOQL). The AOQL of rectifying inspection plan is a very important characteristic. It is possible to design rectifying inspection programs that have specified value of AOQL. The condition for application above basic assumptions of SSTNTVSS $\left(n_{T}, n_{N} ; k\right)$ scheme are the same as that of the SSTNTVSS ( $\mathrm{n} ; \mathrm{k}_{\mathrm{T}}, \mathrm{k}_{\mathrm{N}}$ ) scheme.

The fraction defective in a lot will be

$$
\begin{align*}
& \mathrm{p}=1-\mathrm{F}(\mathrm{v})=\mathrm{F}(-\mathrm{v}) \text { with } \mathrm{v}=(\mathrm{U}-\mu) / \sigma \text { and } \\
& \qquad \mathrm{F}(\mathrm{y})=\int_{-\infty}^{\mathrm{y}} \frac{1}{\sqrt{2 \pi}} e^{-z^{2} / 2} d z \tag{1}
\end{align*}
$$

where $\mathrm{z} \sim \mathrm{N}(0,1)$. Under the $\sigma$ - method plan, the lot would be accepted if $\overline{\mathrm{X}}+\mathrm{k} \sigma \leq \mathrm{U}$, where U is the upper specification limit or $\bar{X}+k \sigma \geq L$, where $L$ is the lower specification limit.

## Operating Procedure

Step 1: Inspect under tightened inspection using the single sampling plan with sample size $\mathrm{n}_{\mathrm{T} \mathrm{\sigma}}$ and the acceptance constant $k$. Accept the individual lot. If $\bar{X}_{T}+k \sigma \leq U$ or $\bar{X}_{T}-k \sigma \geq \mathrm{L}$ where $\overline{\mathrm{X}}$ is the sample mean. If t lots in a row are accepted, switch to normal inspection (Step 2).

Step 2: Inspect under normal inspection using the single sampling plan with sample size $\mathrm{n}_{\mathrm{N} \mathrm{\sigma}}\left(<\mathrm{n}_{\mathrm{T} \mathrm{\sigma}}\right)$ and the acceptance constant $k$. Reject the individual lot. If $\bar{X}_{N}+k \sigma>U$ or $\bar{X}_{N}-k \sigma<L$ where $\bar{X}$ is the sample mean. When an additional lot is rejected in the next $s$ lots after a rejection, switch to tightened inspection.
where $\bar{X}_{T}=\frac{1}{n_{T}} \sum_{\mathrm{i}=1}^{\mathrm{n}_{\mathrm{T}}} \mathrm{X}_{\mathrm{i}}, \overline{\mathrm{X}}_{\mathrm{N}}=\frac{1}{\mathrm{n}_{\mathrm{N}}} \sum_{\mathrm{i}=1}^{\mathrm{n}_{\mathrm{N}}} \mathrm{X}_{\mathrm{i}}$, and $\mathrm{X}_{\mathrm{i}}$ is the measurement of the $\mathrm{i}^{\text {th }}$ unit.
Thus, the SSTNTVS scheme is specified by the parameters $n_{T \sigma}, n_{N \sigma}, k$, $s$ and $t$, where $n_{\sigma}$ and $n_{N}$ are the sample size and k is the acceptance constant of the variables sampling plan under normal and tightened inspection respectively, $t$ is the criterion for switching to normal inspection, and $s$ is the criterion for switching to tightened inspection. The SSTNT scheme with variables sampling scheme is simply designated as SSTNTVSS ( $\left.\mathrm{n}_{\mathrm{T} \sigma}, \mathrm{n}_{\mathrm{N} \sigma} ; \mathrm{k}\right)$.

When $s=4$ and $t=5$, the OC function of the TNTVSS becomes the scheme OC function of MIL - STD 105D that involves tightened and normal inspections was derived by Dodge (1965), Hald and Thyregod (1965) and Stephen and Larson (1967).

## Operating Characteristic function

According to Calvin (1977), the OC function of the TNT scheme is given by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{a}}(\mathrm{p})=\frac{\mathrm{P}_{\mathrm{T}}\left(1-\mathrm{P}_{\mathrm{N}}^{\mathrm{s}}\right)\left(1-\mathrm{P}_{\mathrm{T}}^{\mathrm{t}}\right)\left(1-\mathrm{P}_{\mathrm{N}}\right)+\mathrm{P}_{\mathrm{N}} \mathrm{P}_{\mathrm{T}}^{\mathrm{t}}\left(1-\mathrm{P}_{\mathrm{T}}\right)\left(2-\mathrm{P}_{\mathrm{N}}^{\mathrm{s}}\right)}{\left(1-\mathrm{P}_{\mathrm{N}}^{\mathrm{s}}\right)\left(1-\mathrm{P}_{\mathrm{T}}^{\mathrm{t}}\right)\left(1-\mathrm{P}_{\mathrm{N}}\right)+\mathrm{P}_{\mathrm{T}}^{\mathrm{t}}\left(1-\mathrm{P}_{\mathrm{T}}\right)\left(2-\mathrm{P}_{\mathrm{N}}^{\mathrm{s}}\right)} \tag{2}
\end{equation*}
$$

where $P_{T}$ and $P_{N}$ are the proportion of lots expected to be accepted using tightened ( $n_{T \sigma}, k$ ) and normal ( $n_{N \sigma}, k$ ) variables single sampling plans respectively. Under the assumption of normal distribution, the expression for $\mathrm{P}_{\mathrm{T}}$ and $\mathrm{P}_{\mathrm{N}}$ are given by

$$
\begin{align*}
& \mathrm{P}_{\mathrm{T}} & =\operatorname{Pr}[(\mathrm{U}-\overline{\mathrm{X}}) / \sigma \geq \mathrm{k}]  \tag{3}\\
\text { and } & \mathrm{P}_{\mathrm{N}} & =\mathrm{P}[(\mathrm{U}-\overline{\mathrm{X}}) / \sigma \geq \mathrm{k}] \tag{4}
\end{align*}
$$

respectively. Equations (3) and (4) are substituted in (2) to find $P_{a}(p)$ values for given $p, s, t, n_{T}, n_{N}$ and $k$. As the individual values of X follows normal distribution with mean $\mu$ and variance $\sigma^{2}$, the expressions given in (3) and (4) can be restated as

$$
P_{\mathrm{T}}=\int_{-\infty}^{w_{\mathrm{T}}} \frac{1}{\sqrt{2 \pi}} \mathrm{e}^{\left(-\mathrm{z}^{2} / 2\right)} \mathrm{dz} \text { and } \mathrm{P}_{\mathrm{N}}=\int_{-\infty}^{w_{\mathrm{N}}} \frac{1}{\sqrt{2 \pi}} \mathrm{e}^{\left(-\mathrm{z}^{2} / 2\right)} \mathrm{dz}
$$

respectively, with

$$
w_{T}=\sqrt{n_{\sigma}}(U-k \sigma-\mu) / \sigma=(v-k) \sqrt{n_{\sigma}} \quad w_{N}=\sqrt{n_{\sigma}}(U-k \sigma-\mu) / \sigma=(v-k) \sqrt{n_{\sigma}} \text { and } v=(U-\mu) / \sigma
$$

## SSTNTVSS $\left(\mathbf{n}_{T \sigma,} \mathbf{n}_{\mathrm{N} \sigma} ; \mathbf{k}_{\sigma}\right)$ with known $\sigma$ variables plan as the reference plan

The Six Sigma Tightened - Normal - Tightened Variables Sampling Scheme with known $\sigma$ variables plan as the reference plan has following operating procedure

## Variable Sampling Plan and SSAOQL procedures

If the quality of the accepted lot is p and all defective units found in the rejected lots are replaced by nondefective units in a rectifying inspection plan, the Six Sigma average outgoing quality (SSAOQ) can be approximated as

$$
\begin{equation*}
\mathrm{SSAOQ}=\mathrm{pP}_{\mathrm{a}}(\mathrm{p}) \tag{5}
\end{equation*}
$$

Where $\mathrm{P}_{\mathrm{a}}(\mathrm{p})$ is defined in the equation (2). If $\mathrm{p}_{\mathrm{m}}$ is the proportion nonconforming items at which SSAOQ is maximum, one has

$$
\begin{equation*}
\mathrm{SSAOQL}=\mathrm{p}_{\mathrm{m}} \mathrm{P}_{\mathrm{a}}\left(\mathrm{p}_{\mathrm{m}}\right) \tag{6}
\end{equation*}
$$

If SSAQL $\left(p_{1}\right)$ is prescribed, then the corresponding value of $v_{S S A Q L}$ or $v_{1}$ will be fixed and if $P_{a}(p)$ is fixed at $99.99966 \%$, that is $(1-\alpha)$. Where, $\alpha=0.0000034 \times 10^{-6}$. Hence we have

$$
\begin{equation*}
\mathrm{P}_{\mathrm{a}}\left(\mathrm{p}_{1}\right)=(1-\alpha) \tag{7}
\end{equation*}
$$

which is obtained from equation (2), and $\mathrm{v}_{1}$ satisfies $\mathrm{w}_{\mathrm{N}}=\left(\mathrm{v}_{1}-\mathrm{k}_{\sigma}\right) \sqrt{\mathrm{n}_{\mathrm{N} \sigma}}$ and $\mathrm{w}_{\mathrm{T}}=\left(\mathrm{v}_{1}-\mathrm{k}_{\sigma}\right) \sqrt{\mathrm{n}_{\mathrm{T} \sigma}}$
So that for given values of $n_{\sigma}, w_{N}, w_{T}$ and SSAQL, $k_{N \sigma}, k_{T \sigma}$ are determined.

## Selection of known $\sigma$ SSTNTVSS( $n_{T \sigma}, \mathbf{n}_{\mathrm{N} \sigma} ; \mathbf{k}_{\boldsymbol{\sigma}}$ ) for given SSAQL and SSAOQL

Table 1 is used for selection of $\sigma-$ method SSTNTVSS. For example, if the SSAQL is fixed at $p_{1}=0.00006$ and the SSAOQL is fixed at 0.00007 and $\mathrm{m}=2$, Table 1 yields $\mathrm{n}_{\mathrm{N} \sigma}=2708$, and $\mathrm{k}_{\sigma}=3.92$, which is associated with 4.5 sigma level of SSTNTVSS $\left(\mathrm{n}_{\mathrm{T} \sigma}, \mathrm{n}_{\mathrm{N} \sigma} ; \mathrm{k}_{\sigma}\right)$. The sample size is $\mathrm{n}_{\mathrm{T} \sigma}=\mathrm{m} \mathrm{n}_{\mathrm{N} \sigma}=(2)(2708)=5416$. Thus, for the given IJETRM (http://ijetrm.com/)
requirement, the SSTNTVSS $\left(\mathrm{n}_{\mathrm{T} \sigma}, \mathrm{n}_{\mathrm{N} \mathrm{\sigma}} ; \mathrm{k}_{\sigma}\right)$ is specified by the parameters $\mathrm{n}_{\mathrm{T} \mathrm{\sigma}}=5416, \mathrm{n}_{\mathrm{N} \mathrm{\sigma}}=2708$, and $\mathrm{k}_{\sigma}=3.921$ with 4.5 sigma level.

The user of Table 1 should understand the limitations of plans indexed by SSAOQL. Sampling with rectifying of rejected lots on the one hand reduces the average percentage of nonconforming items in the lots, but on the other hand introduces non-homogeneity in the series of lots finally accepted. That is, any particular lot will have a quality of $\mathrm{p} \%$ or $0 \%$ nonconforming depending on whether the lot is accepted or rectified. Thus the assumption underlying the SSAOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only the average quality matters. For plans listed in Table 1, if the individual lot quality happens to be the product quality $p_{m}$ at which SSAOQL occurs, then the associated probability of acceptance will be poor. Table 2 gives $P_{a}\left(p_{m}\right)$ values of plans given in Table 1. For example, for SSAQL of $p_{1}=0.00005$ and SSAOQL $=0.0001$, Table 2 gives $P_{a}\left(p_{m}\right)=$ 0.69. Then $\mathrm{p}_{\mathrm{m}}=\operatorname{SSAOQL} / \mathrm{P}_{\mathrm{a}}\left(\mathrm{p}_{\mathrm{m}}\right)=0.00015$. In order to avoid such inconvenience, the producer should maintain the process quality more or less at the SSAQL. The high rate of rejection of lots at $p=p_{m}$ will also indirectly put pressure on the producer to improve the submitted quality.

## SSTNTVSS with unknown $\sigma$ variables plan as the reference plan

If the population standard deviation $\sigma$ is unknown, it is estimated from the sample standard deviation $\mathrm{S}(\mathrm{n}-1$ in the division). If the sample size of the unknown sigma variables scheme ( $\mathrm{S}-$ method) is $\mathrm{n}_{\mathrm{Ts}}$ under tightened inspection and $\mathrm{n}_{\mathrm{Ns}}$ under normal inspection and the acceptance constant is k , then the operating procedure is as follows:
Step 1: Inspect under tightened inspection using the single sampling plan with sample size $n_{T s}$ and the acceptance constant $k$. Accept the individual lot. If $\bar{X}_{T}+\mathrm{k} \mathrm{S}_{\mathrm{T}} \leq \mathrm{U}$ or $\bar{X}_{T}-\mathrm{k}_{\mathrm{T}} \geq \mathrm{L}$ where $\overline{\mathrm{X}}$ is a sample mean sample standard deviation S . If t lots in a row are accepted, switch to normal inspection (Step 2).

Step 2: Inspect under normal inspection using the single sampling plan with sample size $\mathrm{n}_{\mathrm{Ns}}$ and the acceptance constant $\mathrm{k}_{\mathrm{N}}$. Reject the individual lot if $\overline{\mathrm{X}}_{\mathrm{N}}+\mathrm{kS} \mathrm{S}_{\mathrm{N}}>\mathrm{U}$ or $\overline{\mathrm{X}}_{\mathrm{N}}-\mathrm{k} \mathrm{S}_{\mathrm{N}}<\mathrm{L}$ where $\overline{\mathrm{X}}$ is a sample mean and sample standard deviation S . When an additional lot is rejected in the next s lots after a rejection, switch to tightened inspection.

Here $\bar{X}$ and $S$ are the average and the standard deviation of quality characteristic respectively from the sample. Under the assumptions for SSTNTVSS stated, the probability of acceptance $P_{a}(p)$ of a lot is given in the equation (2) and $\mathrm{P}_{\mathrm{T}}$ and $\mathrm{P}_{\mathrm{N}}$ respectively are with

$$
w_{T}=\frac{U-k S_{T}-\mu}{S_{T}} \frac{1}{\sqrt{\left(\frac{1}{n_{T s}}+\frac{k^{2}}{2 n_{T s}}\right)}} \quad \text { and } \quad w_{N}=\frac{U-k S_{N}-\mu}{S_{N}} \frac{1}{\sqrt{\left(\frac{1}{n_{N s}}+\frac{k^{2}}{2 n_{N S}}\right)}}
$$

## Selection of unknown $\sigma$ SSTNTVSS( $n_{T \sigma}, \mathbf{n}_{\mathrm{N} \sigma} ; \mathbf{k}_{\boldsymbol{\sigma}}$ ) for given SSAQL and SSAOQL

Table 1 also gives such matched S-method plan. For example, for given SSAQL of $p_{1}=0.00007$ and SSAOQL $=0.0001, \mathrm{~m}=2$, one obtains the parameters of the S -method plan from Table 1 to be $\mathrm{n}_{\mathrm{Ns}}=2509$, and $\mathrm{k}_{\mathrm{s}}=$ 3.808 which is associated with 4.8 sigma level of $\operatorname{SSTNTVSS}\left(\mathrm{n}_{\mathrm{T} \sigma}, \mathrm{n}_{\mathrm{N} \sigma} ; \mathrm{k}_{\sigma}\right)$. The sample size is $\mathrm{n}_{\mathrm{T} \sigma}=m \mathrm{n}_{\mathrm{N} \mathrm{\sigma}}=$ (2) $(6836)=13672$. Thus, for the given requirement, the $\operatorname{SSTNTVSS}\left(\mathrm{n}_{\mathrm{T} \sigma}, \mathrm{n}_{\mathrm{N} \sigma} ; \mathrm{k}_{\sigma}\right)$ is specified by the parameters $\mathrm{n}_{\mathrm{T} \mathrm{\sigma}}=13672, \mathrm{n}_{\mathrm{N} \mathrm{\sigma}}=6836$, and $\mathrm{k}_{\sigma}=3.808$ with 4.8 sigma level.

## Construction of Table $1 \& 2$

For constructing Table 9.4, a trial value of $p_{m}$ is assumed and the probability of acceptance at $p_{m}$ is found using (6) as

$$
\begin{equation*}
\mathrm{P}_{\mathrm{a}}\left(\mathrm{p}_{\mathrm{m}}\right)=\mathrm{SSAOQL} / \mathrm{p}_{\mathrm{m}} \tag{8}
\end{equation*}
$$

The auxiliary variables $\mathrm{v}_{\mathrm{m}}, \mathrm{w}_{\mathrm{Nm}}$ and $\mathrm{w}_{\mathrm{Tm}}$ corresponding to the values of $\mathrm{p}_{\mathrm{m}}$ and $\mathrm{P}_{\mathrm{a}}\left(\mathrm{p}_{\mathrm{m}}\right)$ respectively, are found using (1) to (4). For given values of $p_{1}$, determine the values of $v_{1}, w_{N}$ and $w_{T}$ using the approximation (Abramwitz and Stegun (1972)) for the ordinate of the cumulative normal distribution. With the values of $\mathrm{v}_{\mathrm{m}}, \mathrm{w}_{\mathrm{Nm}}$ and $\mathrm{w}_{\mathrm{Tm}}$, the following equations are derived, which Muthuraj and Senthilkumar(2006) used for calculating $\mathrm{n}_{\sigma}$.

$$
\begin{equation*}
\sqrt{\mathrm{n}_{\sigma}}=(-\mathrm{AOQL}) /\left(\mathrm{p}_{\mathrm{m}}^{2}\left(\mathrm{P}_{\mathrm{T}_{1}}^{\prime} \mathrm{G}+\mathrm{P}_{\mathrm{N}}^{\prime} \mathrm{H}\right) /(\mathrm{A}+\mathrm{B})^{2}\right. \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathrm{P}_{\mathrm{T}_{1}}^{\prime}=-\sqrt{\left(\exp \left(\mathrm{v}_{\mathrm{m}}^{2}-\mathrm{w}_{\mathrm{Tm}}^{2}\right)\right.}  \tag{10}\\
& \mathrm{P}_{\mathrm{N}_{1}}^{\prime}=-\sqrt{\left(\exp \left(\mathrm{v}_{\mathrm{m}}^{2}-\mathrm{w}_{\mathrm{Nm}}^{2}\right)\right.} \tag{11}
\end{align*}
$$

Equation (9) is the formula for finding the sample size of a known $\sigma$ SSTNT Variables Sampling Scheme. With the values of $n_{\sigma}$ obtained from (9), it is then checked to see whether the assumed value of $p_{m}$ corresponds to the proportion non-conforming at which the SSAOQL occurs or not. That is, it is checked to see whether or not the trial value of $\mathrm{p}_{\mathrm{m}}$ satisfies the following condition.

$$
\begin{equation*}
\mathrm{AOQL}+\left[\left(\mathrm{p}_{\mathrm{m}}^{2}\left(\mathrm{P}_{\mathrm{T}_{1}}^{\prime} \mathrm{G}+\mathrm{P}_{\mathrm{N}}^{\prime} \mathrm{H}\right) /(\mathrm{A}+\mathrm{B})^{2}\right]=\mathrm{O}\right. \tag{12}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathbf{P}_{\mathrm{T}_{1}}^{\prime}=-\sqrt{\mathbf{n}_{\mathrm{T}_{\mathrm{\sigma}}} \exp \left(\mathrm{v}_{\mathrm{m}}^{2}-\mathrm{w}_{\mathrm{Tm}}^{2}\right)}=\mathbf{P}_{\mathrm{T}_{\mathrm{i}}}^{\prime}=-\sqrt{\mathrm{mn}_{\mathrm{N}_{\mathrm{\sigma}}} \exp \left(\mathrm{v}_{\mathrm{m}}^{2}-\mathrm{w}_{\mathrm{Tm}}^{2}\right)}  \tag{13}\\
& \mathbf{P}_{\mathrm{N}_{1}}^{\prime}=-\sqrt{\mathbf{n}_{\mathrm{N}_{\sigma}} \exp \left(\mathrm{v}_{\mathrm{m}}^{2}-\mathrm{w}_{\mathrm{Nm}}^{2}\right)} \tag{14}
\end{align*}
$$

The equation (12) was obtained from the following relation

$$
\begin{equation*}
\frac{\mathrm{d}(\mathrm{SSAOQ})}{\mathrm{dp}}=\mathrm{P}_{\mathrm{a}}(\mathrm{p})+\mathrm{p} \frac{\mathrm{dP}_{\mathrm{a}}(\mathrm{p})}{\mathrm{dp}}=0 \tag{15}
\end{equation*}
$$

in which

$$
\begin{equation*}
\frac{\mathrm{d}(\mathrm{SSAOQ})}{\mathrm{dp}}=\frac{\mathrm{P}_{\mathrm{T}}^{\prime} \mathrm{G}+\mathrm{P}_{\mathrm{N}}^{\prime} \mathrm{H}}{(\mathrm{~A}+\mathrm{B})^{2}} \tag{16}
\end{equation*}
$$

If assumed value of $p_{m}$ does not satisfy (12), then another trial value of $p_{m}$ is obtained from (12) by numerical methods. The methods of successive substation is often found to give good results and (12) is rewritten for this purpose as

$$
\begin{equation*}
\mathrm{p}_{\mathrm{m}}=(-\mathrm{AOQL}) /\left[\mathrm{p}_{\mathrm{m}}\left(\mathrm{P}_{\mathrm{T}}^{\prime} \mathrm{G}+\mathrm{P}_{\mathrm{N}}^{\prime} \mathrm{H}\right) /(\mathrm{A}+\mathrm{B})^{2}\right] \tag{17}
\end{equation*}
$$

After determining the next trial value of $p_{m}$, again the values of $\mathrm{v}_{\mathrm{m}}, \mathrm{w}_{\mathrm{Nm}}, \mathrm{w}_{\mathrm{Tm}}$ and $\mathrm{n}_{\sigma}$ are found and the condition (12) rechecked. This iterative procedure continues until the convergence of $p_{m}$ is achieved. Then the value of $k_{\sigma}$ is obtained from (3) and (4).

For obtaining the values of $\mathrm{v}_{1}, \mathrm{w}_{\mathrm{N}}$ and $\mathrm{w}_{\mathrm{T}}$, the approximation for the ordinate of the cumulative normal distribution available in Abramowitz and Stegun (1972) was used.

The S-method plans matching the $\sigma$-method plans were obtained using computer search routine through C++ programme. For selected combinations of SSAQL and SSAOQL, Table $1 \& 2$ was constructed following the above iterative procedure.

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Table 1: SSQSVSS with known and unknown $\sigma$ indexed by SSAQL and SSAOQL $\left(\mathbf{n}_{\mathrm{T} \mathrm{\sigma} \sigma}=\mathbf{m n}_{\mathrm{N} \sigma}\right.$, where $\mathbf{m}=2$ )

| SSAQL( $p_{1}$ ) | SSAOQL | $\mathbf{n}_{\text {T } \sigma}$ | $\mathbf{n}_{\mathbf{N \sigma}}$ | $\mathbf{k}_{\boldsymbol{\sigma}}$ | $\sigma \text { - Level }$ | $\mathbf{n}_{\text {Ts }}$ | $\mathbf{n}_{\text {Ns }}$ | $\mathrm{k}_{\text {s }}$ | $\sigma$ - Level |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00001 | 0.00002 | 3781 | 1890 | 4.313 | 4.3 | 38948 | 19474 | 4.313 | 5.0 |
|  | 0.00003 | 2429 | 1214 | 4.300 | 4.2 | 24884 | 12442 | 4.300 | 4.9 |
|  | 0.00004 | 2158 | 1079 | 4.285 | 4.2 | 21967 | 10983 | 4.285 | 4.9 |
|  | 0.00005 | 1824 | 912 | 4.273 | 4.1 | 18477 | 9238 | 4.273 | 4.8 |
|  | 0.00006 | 1459 | 730 | 4.262 | 4.0 | 14710 | 7355 | 4.262 | 4.7 |
|  | 0.00007 | 1063 | 532 | 4.247 | 3.9 | 10650 | 5325 | 4.247 | 4.7 |
|  | 0.00008 | 636 | 318 | 4.232 | 3.7 | 6330 | 3165 | 4.232 | 4.5 |
|  | 0.00009 | 610 | 305 | 4.219 | 3.7 | 6035 | 3018 | 4.219 | 4.5 |
|  | 0.0001 | 583 | 292 | 4.204 | 3.7 | 5737 | 2869 | 4.204 | 4.5 |
|  | 0.0002 | 521 | 260 | 4.189 | 3.6 | 5090 | 2545 | 4.189 | 4.4 |
|  | 0.0003 | 468 | 234 | 4.174 | 3.6 | 4545 | 2273 | 4.174 | 4.4 |
| 0.00002 | 0.00003 | 3556 | 1778 | 4.306 | 4.3 | 36529 | 18265 | 4.306 | 5.0 |
|  | 0.00004 | 3154 | 1577 | 4.230 | 4.3 | 31373 | 15686 | 4.230 | 5.0 |
|  | 0.00005 | 2659 | 1330 | 4.215 | 4.2 | 26281 | 13141 | 4.215 | 4.9 |
|  | 0.00006 | 1605 | 803 | 4.203 | 4.1 | 15781 | 7891 | 4.203 | 4.8 |
|  | 0.00007 | 1170 | 585 | 4.191 | 3.9 | 11442 | 5721 | 4.191 | 4.7 |
|  | 0.00008 | 700 | 350 | 4.176 | 3.8 | 6800 | 3400 | 4.176 | 4.5 |
|  | 0.00009 | 671 | 335 | 4.161 | 3.7 | 6476 | 3238 | 4.161 | 4.5 |
|  | 0.0001 | 642 | 321 | 4.148 | 3.7 | 6162 | 3081 | 4.149 | 4.5 |
|  | 0.0002 | 573 | 286 | 4.133 | 3.7 | 5466 | 2733 | 4.133 | 4.5 |
|  | 0.0003 | 515 | 257 | 4.118 | 3.6 | 4880 | 2440 | 4.118 | 4.4 |
|  | 0.0004 | 459 | 230 | 4.103 | 3.6 | 4326 | 2163 | 4.103 | 4.4 |
| 0.00003 | 0.00004 | 8516 | 4258 | 4.236 | 4.6 | 84919 | 42460 | 4.236 | 5.2 |
|  | 0.00005 | 3058 | 1529 | 4.160 | 4.3 | 29517 | 14758 | 4.160 | 4.9 |
|  | 0.00006 | 1846 | 923 | 4.145 | 4.1 | 17699 | 8850 | 4.145 | 4.8 |
|  | 0.00007 | 1345 | 672 | 4.132 | 4.0 | 12828 | 6414 | 4.132 | 4.7 |
|  | 0.00008 | 805 | 402 | 4.121 | 3.8 | 7635 | 3817 | 4.121 | 4.6 |
|  | 0.00009 | 771 | 386 | 4.105 | 3.8 | 7270 | 3635 | 4.106 | 4.6 |
|  | 0.0001 | 738 | 369 | 4.090 | 3.8 | 6909 | 3454 | 4.090 | 4.5 |
|  | 0.0002 | 659 | 329 | 4.078 | 3.7 | 6136 | 3068 | 4.078 | 4.5 |
|  | 0.0003 | 592 | 296 | 4.062 | 3.7 | 5477 | 2738 | 4.062 | 4.5 |


|  | 0.0004 | 528 | 264 | 4.047 | 3.7 | 4854 | 2427 | 4.047 | 4.4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.0005 | 428 | 214 | 4.033 | 3.6 | 3909 | 1955 | 4.033 | 4.4 |

Table 1 (continued...)

| SSAQL( $\mathrm{p}_{1}$ ) | SSAOQL | $\mathbf{n}_{\text {T } \sigma}$ | $\mathbf{n}_{\mathrm{N} \sigma}$ | $\mathbf{k}_{\sigma}$ | $\boldsymbol{\sigma}$ - Level | $\mathbf{n}_{\text {Ts }}$ | $\mathbf{n}_{\text {Ns }}$ | $\mathbf{k}_{\text {s }}$ | $\sigma$ - Level |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00004 | 0.00005 | 10475 | 5237 | 4.090 | 4.7 | 98071 | 49036 | 4.090 | 5.3 |
|  | 0.00006 | 3670 | 1835 | 4.074 | 4.3 | 34130 | 17065 | 4.074 | 5.0 |
|  | 0.00007 | 2049 | 1024 | 4.062 | 4.2 | 18953 | 9476 | 4.062 | 4.8 |
|  | 0.00008 | 1493 | 746 | 4.050 | 4.0 | 13738 | 6869 | 4.050 | 4.7 |
|  | 0.00009 | 893 | 447 | 4.035 | 3.9 | 8163 | 4081 | 4.035 | 4.6 |
|  | 0.0001 | 856 | 428 | 4.020 | 3.8 | 7771 | 3886 | 4.020 | 4.6 |
|  | 0.0002 | 819 | 409 | 4.007 | 3.8 | 7392 | 3696 | 4.007 | 4.6 |
|  | 0.0003 | 731 | 366 | 3.992 | 3.8 | 6557 | 3278 | 3.992 | 4.5 |
|  | 0.0004 | 657 | 329 | 3.976 | 3.8 | 5852 | 2926 | 3.976 | 4.5 |
|  | 0.0005 | 586 | 293 | 3.962 | 3.7 | 5189 | 2594 | 3.962 | 4.5 |
| 0.00005 | 0.00006 | 7532 | 3766 | 4.004 | 4.6 | 67894 | 33947 | 4.004 | 5.2 |
|  | 0.00007 | 3494 | 1747 | 3.991 | 4.3 | 31321 | 15661 | 3.991 | 5.0 |
|  | 0.00008 | 1851 | 926 | 3.979 | 4.1 | 16507 | 8254 | 3.979 | 4.8 |
|  | 0.00009 | 1107 | 554 | 3.964 | 3.9 | 9807 | 4903 | 3.964 | 4.7 |
|  | 0.0001 | 1061 | 531 | 3.950 | 3.9 | 9340 | 4670 | 3.950 | 4.6 |
|  | 0.0002 | 1015 | 508 | 3.937 | 3.9 | 8883 | 4442 | 3.937 | 4.6 |
|  | 0.0003 | 907 | 453 | 3.921 | 3.9 | 7878 | 3939 | 3.921 | 4.6 |
|  | 0.0004 | 815 | 407 | 3.906 | 3.8 | 7030 | 3515 | 3.906 | 4.6 |
|  | 0.0005 | 727 | 364 | 3.892 | 3.8 | 6233 | 3116 | 3.892 | 4.5 |
| 0.00006 | 0.00007 | 5416 | 2708 | 3.921 | 4.5 | 47042 | 23521 | 3.921 | 5.1 |
|  | 0.00008 | 2125 | 1063 | 3.909 | 4.2 | 18360 | 9180 | 3.909 | 4.8 |
|  | 0.00009 | 1273 | 637 | 3.893 | 4.0 | 10925 | 5463 | 3.893 | 4.7 |
|  | 0.0001 | 1218 | 609 | 3.879 | 4.0 | 10385 | 5193 | 3.879 | 4.7 |
|  | 0.0002 | 1166 | 583 | 3.866 | 4.0 | 9877 | 4938 | 3.866 | 4.7 |
|  | 0.0003 | 1041 | 520 | 3.850 | 3.9 | 8758 | 4379 | 3.851 | 4.6 |
|  | 0.0004 | 935 | 468 | 3.835 | 3.9 | 7814 | 3907 | 3.835 | 4.6 |
|  | 0.0005 | 835 | 417 | 3.821 | 3.9 | 6927 | 3463 | 3.821 | 4.6 |
| 0.00007 | 0.00008 | 5015 | 2508 | 3.838 | 4.5 | 41954 | 20977 | 3.838 | 5.1 |
|  | 0.00009 | 1732 | 866 | 3.822 | 4.1 | 14384 | 7192 | 3.822 | 4.8 |
|  | 0.0001 | 1657 | 829 | 3.808 | 4.1 | 13672 | 6836 | 3.808 | 4.8 |
|  | 0.0002 | 1585 | 793 | 3.795 | 4.1 | 13001 | 6500 | 3.795 | 4.7 |


|  | 0.0003 | 1416 | 708 | 3.780 | 4.1 | 11532 | 5766 | 3.780 | 4.7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.0004 | 1272 | 636 | 3.765 | 4.0 | 10288 | 5144 | 3.765 | 4.7 |

Table 1 (continued...)

| SSAQL( $\mathrm{p}_{1}$ ) | SSAOQL | $\mathbf{n}_{\text {T } \sigma}$ | $\mathbf{n}_{\text {N }}$ | $\mathbf{k}_{\text {\% }}$ | $\sigma$ - Level | $\mathbf{n}_{\text {Ts }}$ | $\mathbf{n}_{\text {Ns }}$ | $\mathbf{k}_{\text {s }}$ | $\sigma$ - Level |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00007 | 0.0005 | 1135 | 568 | 3.750 | 4.0 | 9118 | 4559 | 3.750 | 4.6 |
| 0.00008 | 0.00009 | 2754 | 1377 | 3.752 | 4.3 | 22140 | 11070 | 3.752 | 4.9 |
|  | 0.0001 | 1842 | 921 | 3.738 | 4.1 | 14708 | 7354 | 3.738 | 4.8 |
|  | 0.0002 | 1762 | 881 | 3.725 | 4.1 | 13984 | 6992 | 3.725 | 4.8 |
|  | 0.0003 | 1574 | 787 | 3.710 | 4.1 | 12403 | 6201 | 3.710 | 4.7 |
|  | 0.0004 | 1414 | 707 | 3.694 | 4.1 | 11063 | 5532 | 3.694 | 4.7 |
|  | 0.0005 | 1262 | 631 | 3.680 | 4.0 | 9804 | 4902 | 3.680 | 4.7 |
| 0.00009 | 0.0001 | 2208 | 1104 | 3.667 | 4.2 | 17053 | 8526 | 3.667 | 4.8 |
|  | 0.0002 | 1958 | 979 | 3.654 | 4.2 | 15025 | 7513 | 3.654 | 4.8 |
|  | 0.0003 | 1748 | 874 | 3.639 | 4.1 | 13324 | 6662 | 3.639 | 4.8 |
|  | 0.0004 | 1571 | 786 | 3.623 | 4.1 | 11883 | 5941 | 3.623 | 4.7 |
|  | 0.0005 | 1402 | 701 | 3.609 | 4.1 | 10529 | 5264 | 3.609 | 4.7 |

Table 2: $P_{a}\left(p_{m}\right)$ Values of known $\sigma$ plans

| SSAOQL | SSAQL ( $\mathbf{p}_{1}$ ) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.00001 | 0.00002 | 0.00003 | 0.00004 | 0.00005 | 0.00006 | 0.00007 | 0.00008 | 0.00009 |
| 0.00002 | 0.90 |  |  |  |  |  |  |  |  |
| 0.00003 | 0.87 | 0.91 |  |  |  |  |  |  |  |
| 0.00004 | 0.86 | 0.88 | 0.91 |  |  |  |  |  |  |
| 0.00005 | 0.85 | 0.85 | 0.88 | 0.93 |  |  |  |  |  |
| 0.00006 | 0.80 | 0.83 | 0.86 | 0.90 | 0.92 |  |  |  |  |
| 0.00007 | 0.71 | 0.74 | 0.77 | 0.81 | 0.83 | 0.85 |  |  |  |
| 0.00008 | 0.63 | 0.66 | 0.69 | 0.73 | 0.75 | 0.77 | 0.81 |  |  |
| 0.00009 | 0.62 | 0.65 | 0.68 | 0.72 | 0.74 | 0.76 | 0.80 | 0.83 |  |
| 0.0001 | 0.57 | 0.60 | 0.63 | 0.67 | 0.69 | 0.71 | 0.75 | 0.78 | 0.82 |
| 0.0002 | 0.51 | 0.54 | 0.57 | 0.61 | 0.63 | 0.65 | 0.69 | 0.72 | 0.76 |
| 0.0003 | 0.48 | 0.51 | 0.54 | 0.58 | 0.60 | 0.62 | 0.66 | 0.69 | 0.73 |
| 0.0004 |  | 0.48 | 0.51 | 0.56 | 0.57 | 0.60 | 0.63 | 0.67 | 0.70 |
| 0.0005 |  |  | 0.46 | 0.51 | 0.52 | 0.55 | 0.58 | 0.62 | 0.65 |

