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RP-120: FORMULATION OF SOLUTIONS OF STANDARD BIQUADRATIC CONGRUENCE OF EVEN COMPOSITE MODULUS

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ABSTRACT

In this paper, a standard solvable bi-quadratic congruence of even composite modulus is considered for study. It is discussed fully and its solutions are formulated. Now, it becomes possible to find all the solutions directly and very easily. No discussion is found in the earlier literature of mathematics. A formulation is established first time for the solutions of the congruence under consideration. This is the merit of the paper.

Key-words: Bi-quadratic congruence, Binomial expansion, Formulation

INTRODUCTION

Bi-quadratic congruence is a neglected topic in Number Theory as the cubic congruence. Only quadratic congruence is considered as a prominent topic in it and much had been discussed and studied earlier. Euler, Gauss and Fermat had given much attention to the topic of standard quadratic congruence. The author found a vast gap in the literature for research and has started his journey to study the standard cubic congruence and also the standard bi-quadratic congruence for formulation.

LITERATURE-REVIEW

A standard bi-quadratic congruence is seldom studied and is an unseen topic of Mathematics (Number Theory). Nothing is seen in the literature of mathematics. Only a definition of bi-quadratic residue is mentioned. Also the solvability condition is not found. It is said that if $x^4 \equiv a \pmod{p}$ is solvable, then a is called the bi-quadratic residue of p **[6].** Earlier mathematicians approached less to the topic.

The author first time tried his best to formulate some standard bi-quadratic congruence successfully such as:

$x^4 \equiv a^4 \pmod{4.p^n}$

& $x^4 \equiv a^4 \pmod{8.p^n}$;

$x^4 \equiv a^4 \pmod{2^m.p^n}$;

$x^4 \equiv a^4 \pmod{4^n.b}$.

p being a positive prime integer; n any positive integer **[1], [2], [3], [4], [5].**

In continuation of the above research, the author considered the next research paper:

$x^4 \equiv a^4 \pmod{2^m}; m \geq 4$ for formulation and studied.

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NEED OF RESEARCH

No formulation of the said congruence under consideration is found in the earlier literature of mathematics and it is difficult to find all the solutions. No existed method is also found.
It is very difficult to find the solutions of the congruence. To get rid of such difficulty, the author considered the congruence for study and tried his best to formulate and presented his sincere efforts in this paper. To have a formula for solutions is the need of the research.

PROBLEM-STATEMENT

Here the problem is:
"To formulate the standard solvable bi-quadratic congruence of even
Composite modulus of the type: $x^4 \equiv a^4 \ (mod\ 2^m); m \geq 4$.

ANALYSIS & RESULT

Consider the said congruence: $x^4 \equiv a^4 \ (mod\ 2^m); m \geq 4$.

Let us consider that $x = (2^{m-2}k \pm a); k = 0, 1, 2, \ldots\ldots\ldots\ldots$

Then, $x^4 = (2^{m-2}k \pm a)^4$

$$= (2^{m-2}k)^4 \pm 4.(2^{m-2}k)^3.a + \frac{4.3}{1.2}(2^{m-2}k)^2.a^2 \pm \frac{4.3.2}{1.2.3}(2^{m-2}k)^1.a^3 + a^4$$

$$= a^4 + 2^m k(t); t \ a\ positive\ integer.$$

$$\equiv a^4 (mod\ 2^m).$$

Therefore, $x \equiv (2^{m-2}k \pm a)$ (mod 2^m) are the solutions of the said congruence.

For $k = 4$, $the\ solution\ formla\ becomes\ x \equiv (2^{m-2}.4 \pm a)\ (mod\ 2^m)$

$$\equiv (2^m \pm a)\ (mod\ 2^m)$$

$$\equiv \pm a\ (mod\ 2^m),$$ which is the same solution as for $k = 0$.

For $k = 5 = 4 + 1 = 2^2 + 1$, then the solution formula becomes

$$x = 2^{m-2}.(2^2 + 1) \pm a\ (mod\ 2^m)$$

$$\equiv \{2^{m-2}.2^2 + 2^{m-2}.1\} \pm a\ \ (mod\ 2^m)$$

$$\equiv \{2^m + 2^{m-2}\} \pm a)\ (mod\ 2^m)$$

$$\equiv (2^{m-2} \pm a)\ (mod\ 2^m)$$

Which is the same solution as for k=1.

Similarly for $k = 6, 7,$ 8, the solutions are the same as for $k = 2, 3, 0$.

Thus, all the solutions are given by $x \equiv (2^{m-2}.k \pm a)\ (mod\ 2^m); k - 0, 1, 2, 3.$

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Therefore, the congruence has exactly eight solutions.

Sometimes, the bi-quadratic congruence can be given as $x^4 \equiv b \ (mod\ 2^m)$.

If $b = a^4$, then nothing remains to prove. But if $b \neq a^4$, then the congruence is solved as under: $x^4 \equiv b + l.2^m = a^4 \ (mod\ 2^m)$ for a suitable l, a positive integer. Then, the congruence can be solved as above.

ILLUSTRATIONS

Consider the congruence $x^4 = 1 \ (mod\ 16)$.

It can be written as $x^4 \equiv 1^4 \ (mod\ 2^4)$.

It is of the type $x^4 \equiv a^4 (mod\ 2^m) \quad with\ a = 1, \ m = 4.$

The solutions are given by $x \equiv (2^{m-2}k \pm a) \ (mod\ 2^m)$.

$$\equiv (2^{4-2}.k \pm 1) \ (mod\ 2^4)$$

$$\equiv (4.k \pm 1) \ (mod\ 2^4); k = 0, 1, 2, 3.$$

$$\equiv \pm 1; 4 \pm 1, ; 8 \pm 1; 12 \pm 1 \ (mod\ 16)$$

$$\equiv 1, 15; 3, 5; 7, 9; 11, 13 \ (mod\ 16).$$

Consider the congruence $x^4 \equiv 17 \ (mod\ 64)$.

It can be written as $x^4 \equiv 17 + 64 = 81 = 3^4 \ (mod\ 2^6)$.

It is of the type $x^4 \equiv a^4 \ (mod\ 2^m) \quad with\ a = 3, m = 6.$

Hence all the solutions are given by

$$x \equiv (2^{m-2}.k \pm a) \ (mod\ 2^m).$$

$$\equiv (2^4 k \pm 3)(mod\ 2^6).$$

$$\equiv (16k \pm 3) \ (mod\ 64)$$

$$\equiv 0 \pm 3; 16 \pm 3; \ 32 \pm 3; \ 48 \pm 3 \ (mod\ 64)$$

$$\equiv 3, 61; 13, 19; 29, 35; 45, 51 \ (mod\ 64).$$

Let us consider one more example: $x^4 \equiv 256 (mod\ 512)$.

It can be written as: $x^4 \equiv 4^4 (mod\ 2^9)$.

Here, $a = 4, m = 9$.

The solutions are given by $x = (2^{m-2}.k \pm a) \ (mod\ 2^m); k = 0, 1, 2, 3.$

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$$\equiv (2^{9-2}.k \pm 4)\ (mod\ 2^9).$$

$$\equiv (2^7.k \pm 4)\ (mod\ 2^9).$$

$$= (128.k \pm 4)\ (mod\ 512).$$

$$\equiv (0 \pm 4); (128\ \pm 4); (256 \pm 4); (384 \pm 4)\ (mod\ 512).$$

$$\equiv 4, 508; 124, 132; 252, 260;\ 380, 388\ (mod\ 512).$$

These are the required eight incongruent solutions of the congruence.

CONCLUSION

Therefore, it can be concluded that the standard solvable bi-quadratic congruence under consideration $x^4 \equiv a^4 (mod\ 2^m); m \geq 4$ has exactly eight incongruent solutions which are given by: $x \equiv (2^{m-2}.k \pm a)\ (mod\ 2^m); k = 0, 1, 2, 3.$

MERIT OF PAPER

In this paper, the standard solvable bi-quadratic congruence is studied and formulated. It now becomes easy to find all the solutions directly. Thus, formulation of the solutions of the congruence is the merit of the paper.

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