

**DIVERSIFICATION STRATEGY FOR OPTIMIZING RISK OF PORTFOLIO**

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**ABSTARCT**

Diversification is a strategic choice that investors use to optimize risk of portfolio. It is an opportunity by which investors improve from his micro-firm into macro-firm. The investors' aim is to make an optimal choice that leads to minimization of risk and maximization of return, but the procedures that lead to these objectives are not easily achieved. The purpose of this study is to propose diversification strategy that leads to minimization of risk and maximization of return. In addition, this study also investigates impacts of each asset in minimizing risk of portfolio, in order to have knowledge of the strength of each asset in risk reduction rate of the portfolio. The assets allocations divulge by Black Litterman model are used to estimate risk of both portfolios and assets. We explore DataStream (Yahoo finance) of Gold, Oil, Silver and Platinum spans from 1<sup>st</sup> January, 2000 to 1<sup>st</sup> September, 2016. It is observed that investing on Gold minimizes higher risk and achieve more benefits than other assets in the portfolio. In view of these facts, it means diversifying in gold acts as hedge/safe haven for investors during economic crisis.

**Keywords:** Portfolio, Diversification, Black Litterman, Investment, Asset

**INTRODUCTION**

Diversification is investing in many assets in order to minimize risk or maximize return in the portfolio. It is an opportunity by which investors develop from his small firm into other market products (Badertscher, Shroff, & White, 2013). Study on diversification has caught the attention of many management scholars and is one of the vital areas of study in business. Among others, researchers have studied the antecedents of diversification and the financial performance (Elango, Ma, & Pope, 2008). Investors indeed would explore the benefit of diversification by investing on 10 to 15 securities as suggested by scholars of financial management. The benefit of investing in a large number of securities was visibly established in a more recent study (Fragkiskos et al., 2014)

Diversification is an approach by which firm multiply from its main business into other product market (Hsu, Chen, & Cheng, 2013). Study reveals that corporate management strongly involved diversification activities and many scholars established this fact. Diversification advances debt capacity, reduce the chances of bankruptcy by introducing new products/markets (Higgins & Schall, 2016) and improves asset placement and productivity. A diversified firm can move funds from a cash surplus unit to a deficit unit without taxes or transaction costs. Diversified firms pool unsystematic risk and reduce the variability of operating cash flow enjoy comparative benefit in hiring key employees to have a higher sense of job security (Nyaingiri & Ogollah, 2015).

This study concentrates on procedures of diversification that optimizes risk of portfolio and also to estimate the effects of each asset in minimizing risk of portfolios. The first is we partition the assets into four portfolios with the aid of Black Litterman (BL) allocations and use weight on investor views  $\tau = 1.0$  which is considered to yield the best allocation (Satchell, 2000; Walters, 2013), with this allocations we shall estimate the risk of portfolios and assets, hence take decision on optimal portfolio. Furthermore, we shall identify and discard redundant asset(s) for profitable asset(s)

The remaining parts of this paper is organized as follow: section two reviews literatures, section three discusses the data used, section four explains the methodology adopted, section five discusses the findings and section six concludes the paper.

**Literature Reviews**

Modern Portfolio theory is a finance theory that attempts to minimize risk of the portfolio and maximize portfolio expected return. Harry Markowitz (1952) was the first to discover the theory of modern portfolio. His discovery was filled with insights and ideas that anticipated many of the subsequent growth in the field. He originated a portfolio problem as a choice of the mean variance portfolio of assets. He observed that risk encountering by investors was portfolio risk which would lead to a basic and important point that the risk of a stock should not only be estimated just by the variance of the stock but also by the covariance. Moreover, he also mentioned that the best (optimal) portfolio should consist of assets that are perfectly negatively correlated. He noted that there are many perfectly positively correlated assets in circulation. This observation gives rise to the theory of diversification (Markowitz, 1952, 1959).

The most important aspect of Markowitz model was his description of the impact on portfolio diversification by the number of securities within a portfolio and their covariance relationships (Mangram, 2013). They used data on sectoral level of employment and value added to generate new and robust evidence that economic growth through stages of diversification and that sectoral concentration follows a U-shaped pattern in relation to per capital income (Imbs & Wacziarg, 2003). It is observed that mean-variance (MV) optimization is still the best theory of portfolio optimization but it is difficult to implement in practice. The asset weights are extremely sensitive to inputs and the inputs are difficult to achieve. Furthermore through MV it is not possible for investors to express their opinions on relative asset performance and their confidence in their selected expected asset returns

Black and Litterman improved on the original MV model by combining mean-variance optimization of Markowitz and CAPM (Black & Litterman, 1991). The original model was first developed in 1990 and a year later they elaborate on the strategic asset allocation that is embedded with investor's views in a global sense. The model does not consider the assumption that expected returns are always at equilibrium with CAPM. Rather as expected returns deviate from the mean, imbalances in the markets will attempt to drive them back. Therefore, it is observed that investors would make more returns by combining their views about returns with the information in the equilibrium (Black & Litterman, 1991)

Moreover, additional vital feature of the BL structure is that investors should be willing to take risk according to their views and this should be done when they have strong evidence to support their views (Bevan & Winkelmann, 1998). Black Litterman model (BLM) uses Bayesian approach to syndicate the views from the investor with respect to the expected returns of one or more assets with the market equilibrium vector of expected returns to provide a new, mixed estimate of expected returns. The new vector of returns results to intuitive portfolio gives a reasonable portfolio weight (Idzorek, 2005b). Hence, the model produces better stable result than classical mean-variance optimization.

Finally, in our study we used EGARCH to estimate the two parameters of BLM; investor views and level of uncertainty, we adopted to use higher value of weight on investor's views  $\tau = 1.0$ . The data used are gold, oil, silver and platinum.

### Methodology

The methodology adopted is Mean-Variance Model.

Consider the following minimization constraint:

$$\begin{aligned} \min \frac{1}{2} \sigma_p^2 &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \\ &= \frac{1}{2} (w' \Omega w) \end{aligned} \quad (1)$$

where  $w' = (w_1, w_2, \dots, w_n)$  and  $\Omega$  is the  $(n \times n)$  covariance matrix  $\{\sigma_{ij}\}$ . The constraints are

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (2)$$

$$\sum_{i=1}^n w_i = 1 \quad (3)$$

For short selling, we use Lagrange multipliers;  $\lambda_1 \dots \lambda_n$  for the constraints

$$\text{Min } L = (1/2) \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} - \lambda_1 \left( \sum_{i=1}^n w_i ER_i - ER_p \right) - \lambda_2 \left( \sum_{i=1}^n w_i - 1 \right) \quad (4)$$

$$L = (1/2) \left[ \begin{array}{l} w_1^2 \sigma_{11} + 2w_1 w_2 \sigma_{12} + 2w_1 w_3 \sigma_{13} + 2w_1 w_4 \sigma_{14} + \\ w_2^2 \sigma_{22} + 2w_2 w_3 \sigma_{23} + 2w_2 w_4 \sigma_{24} + w_3^2 \sigma_{33} + \\ 2w_3 w_4 \sigma_{34} + w_4^2 \sigma_{44} \end{array} \right] - \lambda_1 [w_1 ER_1 + w_2 ER_2$$

$$+ w_3 ER_3 + w_4 ER_4 - ER_p] - \lambda_2 [w_1 + w_2 + w_3 + w_4 - 1]$$

Differentiating equation (5) with respect to  $w_i, \lambda_i$  gives the following first order conditions (FOC)

$$\frac{\partial L}{\partial w_1} = (2w_1 \sigma_{11} + 2w_2 \sigma_{12} + 2w_3 \sigma_{13} + 2w_4 \sigma_{14}) - \lambda_1 ER_1 - \lambda_2 = 0 \quad (6)$$

$$\frac{\partial L}{\partial w_2} = (2w_2 \sigma_{22} + 2w_1 \sigma_{12} + 2w_3 \sigma_{23} + 2w_4 \sigma_{24}) - \lambda_1 ER_2 - \lambda_2 = 0 \quad (7)$$

$$\frac{\partial L}{\partial w_3} = (2w_3 \sigma_{33} + 2w_1 \sigma_{13} + 2w_2 \sigma_{23} + 2w_4 \sigma_{34}) - \lambda_1 ER_3 - \lambda_2 = 0 \quad (8)$$

$$\frac{\partial L}{\partial w_4} = (2w_4 \sigma_{44} + 2w_1 \sigma_{14} + 2w_2 \sigma_{24} + 2w_3 \sigma_{34}) - \lambda_1 ER_4 - \lambda_2 = 0 \quad (9)$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^4 w_i ER_i - ER_p = 0 \quad (10)$$

$$\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^4 w_i - 1 = 0 \quad (11)$$

For minimum variance portfolio

From equations (10) and (11) and generalized to  $n$ -assets case can be written as

$$\sum_{i=1}^n w_i ER_i = ER_p \quad (12)$$

$$\sum_{i=1}^n w_i = 1 \quad (13)$$

where  $\Omega = \sigma_{ij} = (n \times n)$  covariancematrix,  $ER$  is  $(n \times 1)$ ,  $\lambda_i$  are scalars,  $w_i$  are weights of assets,  $\sigma_{ii}$  are variances, and  $\sigma_{ij}$  equation (10) arbitrarily set  $ER_p$  to any fixed value, we have  $(n+2)$  linear equation and  $(n+2)$  unknowns, the  $w_i$  and  $\lambda_i$ . These linear equations are easily solved using Microsoft Excel (spread sheet) to give the optimal weights for one point on the minimum variance portfolio. We estimate expected returns  $ER_i$ , standard deviations and covariances  $\sigma_{ij}$ . Having

obtained the optimal weight  $w_i$  ( $i = 1, 2, \dots, n$ ) these substituted in  $\sigma_p^2 = w' \Omega w$  and  $ER_p = w' ER$  to give one point on the efficient frontier.

A portfolio of  $n$  assets is denoted by a vector  $x \in R^n$  with  $\sum_{i=1}^n x_i = 1$ . Let the returns of an asset  $i$  be denoted by  $\mathfrak{R}_i$  and expected return of asset  $i$  be  $E(\mathfrak{R}_i)$ . Then the expected return vector is  $E(\mathfrak{R}) = \text{col}\{E(\mathfrak{R}_i)\} \in R^n$ , ( $i=1, 2, \dots, n$ ). The covariance matrix is denoted by  $\Sigma \in R^{n \times n}$ . The covariance of assets  $i$  and  $j$  is given as  $\sigma_{ij}$  (Horasanlı & Fidan, 2007). The return  $\mathfrak{R}_p$  of portfolio is estimated by

$$\begin{aligned} \mathfrak{R}_p &= \sum_{i=1}^n x_i \mathfrak{R}_i \\ E(\mathfrak{R}_p) &= E\left(\sum_{i=1}^n x_i \mathfrak{R}_i\right) \\ \sum_{i=1}^n E(x_i \mathfrak{R}_i) &= \sum_{i=1}^n x_i E(\mathfrak{R}_i) \\ &= x' E(\gamma) \end{aligned} \quad (14)$$

The variance of return of the portfolio can be computed as:

$$\begin{aligned} \sigma_p^2 &= \sigma_i^2 \left(\sum_{i=1}^n x_i \mathfrak{R}_i\right) \\ &= \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} (x_i \mathfrak{R}_i, x_j \mathfrak{R}_j) \\ &= \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} (\mathfrak{R}_i \mathfrak{R}_j) \\ &= \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \\ &= x' \Sigma x \end{aligned} \quad (15)$$

The expected return of equilibrium portfolio as:

$$\Pi = \delta \sum x_{mkt} \quad (16)$$

where  $\Pi$  is the expected return of market equilibrium,  $\delta$  is the risk aversion,  $x_{mkt}$  is the market weight.

The improvement in the BLM allows the investors to combine their views directly in the model in an intuitive way. The views can be relative. The views have to be in the same format with constraints. The investors should be able to fix a level of confidence in his views. This requirement may be as follows:

$$P.E(\mathfrak{R}) = Q + \varepsilon \quad (17)$$

where  $P$  is the vector that describes the assets concerned by the views,  $Q$  is the vector of their performances and  $\varepsilon$  is the random normal vector of error terms,  $\varepsilon \sim N(0, \Omega)$  with diagonal variance matrix  $\Omega$ . It is assumed that the market is rotating around an equilibrium point and the same with investors' portfolio in respect to CAPM hypothesis (Hidalgo & Desportes, 2014)

Let the mean  $E(\mathcal{R}) = \Pi$ , the covariance, assumed to be proportional to  $\Sigma$ , with factor of uncertainty  $\tau$ ,  $E(\mathcal{R}) \sim N(\Pi, \tau\Sigma)$ .

The equation below is known as the Black Litterman equation and represents the expected return vectors that is produced from a Bayesian mixing of the implied equilibrium excess return vector ( $\Pi$ ) and the vector of investor views ( $Q$ )

$$E(\mathcal{R}) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \tag{18}$$

### Data

The sample data consists of monthly closing spot prices for Gold, Silver, Platinum, and Oil. The data spans from 1<sup>st</sup>January, 2000 to 1<sup>st</sup>September, 2016 with a total of 200 observations. The data are obtained from DataStream (Yahoo finance).

### Results and Discussion

The assets allocations results divulge from BLM is used for estimation of portfolios risk and assets. The proposed diversification procedures for optimizing risk of portfolios are given as follow: first partitioning of assets into portfolios, second estimation of risk of the portfolios, third calculation of risk of each asset, fourth swapping of redundant asset (lowest risk-reduction asset) for any profitable asset, fifth computation of risk for portfolios with new asset, in order to decide on optimal portfolio. These procedures are used to develop diversified portfolio and estimate risks in Table 3 below.

**Table 3: Assets in Portfolios**

Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Gold	Gold	Gold	Oil
Oil	Oil	Platinum	Silver
Silver	Platinum	Silver	Platinum

**Table 4: Portfolios Risk**

Benchmark Portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Gold	Gold	Gold	Gold	Oil
Oil	Oil	Oil	Platinum	Silver
Silver	Silver	Platinum	Silver	Platinum
Platinum	0.0068	0.0088	0.0078	0.0106
0.0068				

Table 3 displayed portfolios1 to 4 and Table 4 presents portfolios' actual risk w. From the table, Portfolios1 to 4 exhibit 0.0068, 0.0088, 0.0078and 0.0106 risks respectively. It is observed that Porfolio1 has the lowest risk with the values 0.0068 which is the same benchmark portfolio risk and portfolio4 contains highest risk with the values0.0106. Hence it is noted that portfolios with gold divulged minimum risk while portfolio with platinum generated high risk. It implies that the presence of gold in the portfolios minimizes the risk of the portfolio while the presence of platinum made no impact in the portfolio. Moreover, it is noticed that portfolio 1 contains the same risk with benchmark portfolio; this means portfolio 1 is as good as benchmark portfolio. This intrigues us to investigate the strength of each asset in risk reduction. Table 5 presents the strength of each asset.

**Table 5: Assets Risk-Reduction strength**

Gold	Oil	Silver	Platinum
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<b>0.0038</b>	<b>0.0010</b>	<b>0.0020</b>	<b>0</b>
<b>56%</b>	<b>15%</b>	<b>29%</b>	<b>0%</b>

The total risk of benchmark portfolio is 0.0068; this is the extent to which these four assets can reduce the benchmark portfolio. It is also our interest to know the reduction strength of each asset. Table 5, shows that gold, oil, silver and platinum exhibit risk reduction strengths as 0.0038, 0.0010, 0.0020 and 0 respectively. This implies that gold has 56% strength of risk reduction, oil is 15%, and silver has 29% while platinum has 0% in risk reduction. This vividly shows that gold made more impacts in risk reduction than other assets. It is worth stating that gold reduced more than half risk of the portfolios. This is the reason portfolio1 that contained gold and no platinum has lowest risk while portfolios4 with platinum and no gold has highest risk. According to our result platinum makes no impact in risk reduction of the portfolio and this is called redundant asset, therefore it is worth no investing. The last step of our procedures is to discard the redundant asset and swap with valuable asset.

### CONCLUSION

This paper proposed diversification procedures in optimizing risk of portfolio. In addition, we investigate the impacts of each asset we diversify in to know the strength of each asset in risk reduction of portfolios. Diversification is a strategic approach for minimizing risk of portfolio but if not done according to standard procedures, it may not fulfil its purpose. In this study BLM was used for assets allocations being the best assets allocation model in finance at present (Idzorek, 2005a), also, we wish to state that standard deviation is used in measuring the value of risk in this study. The results of BLM were used to estimate both risk exhibits by portfolios and assets. Hence, it is observed that portfolio1 is the optimal portfolio to invest for rational investors. Moreover, it is noticed that portfolio1 has the lowest risk, which is the same as benchmark portfolio risk. This implies that portfolio 1 is as good as benchmark portfolio. This is as a result of the presence of gold and absence of platinum in the portfolio. According to this study, in order to minimize risk of portfolios, there is need for investors to; first partition assets into portfolios second estimate the risk of the partitioned portfolios third estimate the risk of each asset to know the strength of risk reduction of all the assets, fourth swap redundant asset for profitable asset fifth, compute risk for portfolios with new asset, in order to decide on optimal portfolio. Moreover, investors should endeavour to add high risk reduction assets like gold to portfolios and remove redundant asset like platinum in order to minimize portfolio risk. In view of this fact, we wish to state that gold is of great strength in risk reduction that it can serve as hedge and safe haven during financial crisis.

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