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RELIABILITY MODELING AND PERFORMANCE ANALYSIS OF A STANDBY REDUNDANT SYSTEM USING MARKOVIAN BIRTH-DEATH PROCESS

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ABSTRACT
In an industrial system, availability plays and important role in the direction of an industrial growth as the profit is directly related to the production volume which depends upon performance. To achieve high system availability, proper maintenance management system supported by adequate resources as manpower, spares and machines etc. are required. Thus it is a cyclic chain, better the maintenance facilities, higher the system availability higher the production rate and hence, higher the profit. Mathematically, the term availability is used to indicate the probability of a system of equipment being in operating condition at any time. The availability of a system is a combined measure of both reliability and maintainability. In the present work, the availability models have been developed to evaluate the performance of system, busy period of the repair man and profit of the system were evaluated. The mathematical modeling has been done on the basis of certain assumptions. Then these models have been analyzed and optimum system availability, profit values are determined by using Genetic Algorithm which provide different combinations of failure and repair rates of the subsystem. The whole process of development and analysis of some performance evaluating systems industry may facilitate decision making regarding maintenance work in future.

Key Words: Availability, profit, maintainability, reliability, markov process, birth-death.

INTRODUCTION

When the states of systems are probability based, then the model used is a Markov probability model. Several approaches have been used to analyze the system availability e.g. Monte Carlo Simulation approach applied for the extremely complex system to analyze availability, but the cost of experimentation was very high. The Markovian approach was frequently used for the availability analysis taking exponential distribution for failure and repair times while due to mathematical complexities, relatively less work has been done taking distribution other than exponential. A Markov process is a stochastic process in which at any given time, the subsequent courses of process is affected only by the state at the given time and does not depend on the character of the process at any preceding time. Markov models are the functions of two random variables. The state of the system and the time of observation. Availability studies primarily deal with the discrete state, continuous time models. Such a model is characterized by a probability matrix whose typical element p_{ij} denotes the probability of transition from one state i to a mutually exclusive state j.

Little literature can be found dealing with evaluation of availability, busy period of repairman as well as overall profit generated. This research work intends to use profits as a means of measuring the effectiveness of the system. However, availability/profit of an industrial system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. Improving the reliability and availability of system sub-system, the production and associated profit will also increase. Increase in production leads to the increase of profit. This can be achieved high production profit consider. This can be achieved by maintaining a reliability and availability at highest order. To achieve both production and profits, the system should remain operative for maximum possible duration it is important to consider profit as well as the quality requirement.

Kumar *et al.* (1988, 1989, 1990, 1991, and 1992) used markov modeling in the analysis and evaluating the performance of sugar, paper, and fertilizer plants. They carried out the analysis assuming the failure and repair rates of these systems to be constant. Dekker and Grocnendijck (1995) discussed the importance of

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various analytical and simulation techniques for availability modeling and effective assessment of continuous production systems with the main objectives of economic optimization. They emphasized the need for carrying availability analysis at the component level rather than targeting at system level. Arora and Kumar (1997, 2000) carried out stochastic analysis of power generation and coal handling systems in a thermal power plant with markov chains. They studied the performance of the system in three states viz: good, reduced, and failed.

Sarkar (2000) discussed the mathematical method for obtaining availability and limiting average availability of a periodically inspected system which was supported by a spare unit, and maintain with perfect repairs. Lai (2002), Blischke (2003), Yadav *et al.* (2003), and Dai et al. (2003) performed reliability and availability analysis for some complex systems.

OBJECTIVES

The main aim of the research work is the modeling and performance analysis of a standby redundant system using markovian birth-death process. The specific objectives are as follows:

(a) To develop an reliability model for evaluating the performance of various operating subsystems.
(b) To determine the effects of availability on failure and repair rates of the operating subsystems.
(c) To determine the effects profit on failure and repair rates of the operating subsystems.

METHODOLOGY

A typical system consists of a number of subsystems connected to each other logically either in series or in parallel in most cases. The performance of the system depends on the configuration and performance of its subsystems.

Before analyzing the failure data it is better to describe the configuration of the system and classify it into various subsystems so that the failure can be categorized. The system here present consists of five subsystems, A, B, C, D and E. The system model formulation is carried out base on markovian birth-death process using probabilistic approach.

System Structure

The system consists of five dissimilar subsystems which are:

1. Subsystem A: Single unit in series whose failure causes complete failure of the entire system
2. Subsystem B: Single unit in series whose failure causes complete failure of the entire system
3. Subsystem C: Consist of four units in parallel on cold standby. Total failure of the system occurs only when all of the four units have failed
4. Subsystem D: Single unit in series whose failure causes complete failure of the entire system
5. Subsystem E: Single unit in series whose failure causes complete failure of the entire system

5. Notations.

◯ ; This indicates the system is in full working state

▭ ; This indicates the system is in total failure

A, B, C, D and E. represent full working states of the system.

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C1, C2, C3 and C4;denote the systems working on cold standby units.

λ_i; i = 1, 2,3,4,5. ; represent the failure rates of subsystems A, B, C, D, D and E respectively.

μ_i; i = 1,2,3,4,5.; Represent the repair rate of subsystems A, B, C, D and E. respectively

$p_0(t), p_1(t), p_2(t), p_3(t)$; Denote the probabilities of the system working with full capacity at time t.

A_k, k = 4, 5, 6, 7, 8, 9,… 20. Probability of the system in failed states.

$\frac{dp_i}{dt}, i = 0,1,2,3,4,5,...,20$. Represent the derivatives with respect to t.

A_v; Steady state availability

B_p; Busy period of the repair man

P_F; Profit

$X_i = \frac{\lambda_i}{\mu_i}$

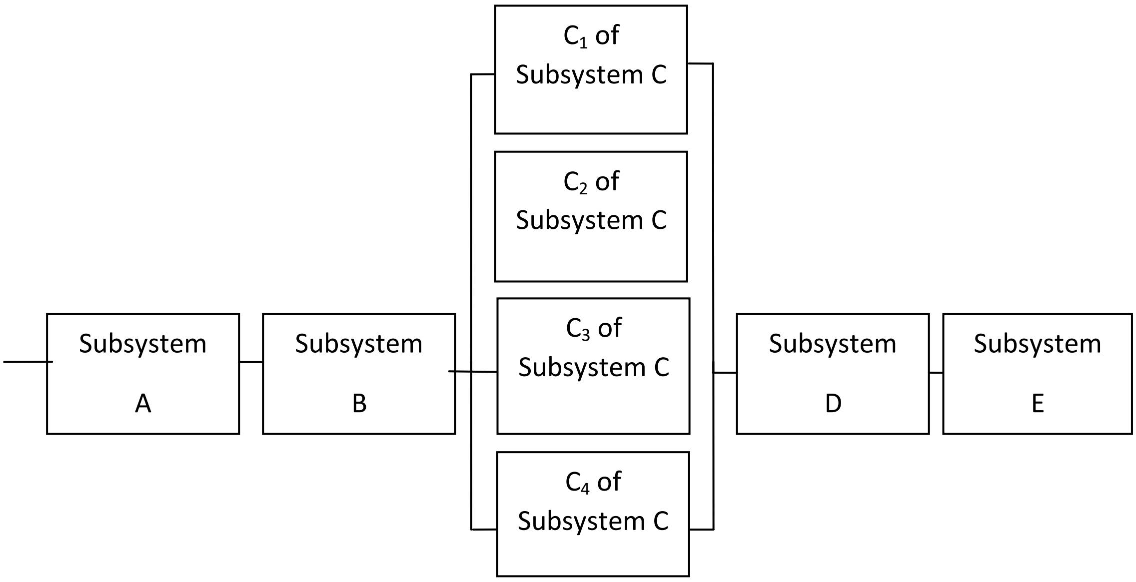

Fig.1 Block Diagram of the system

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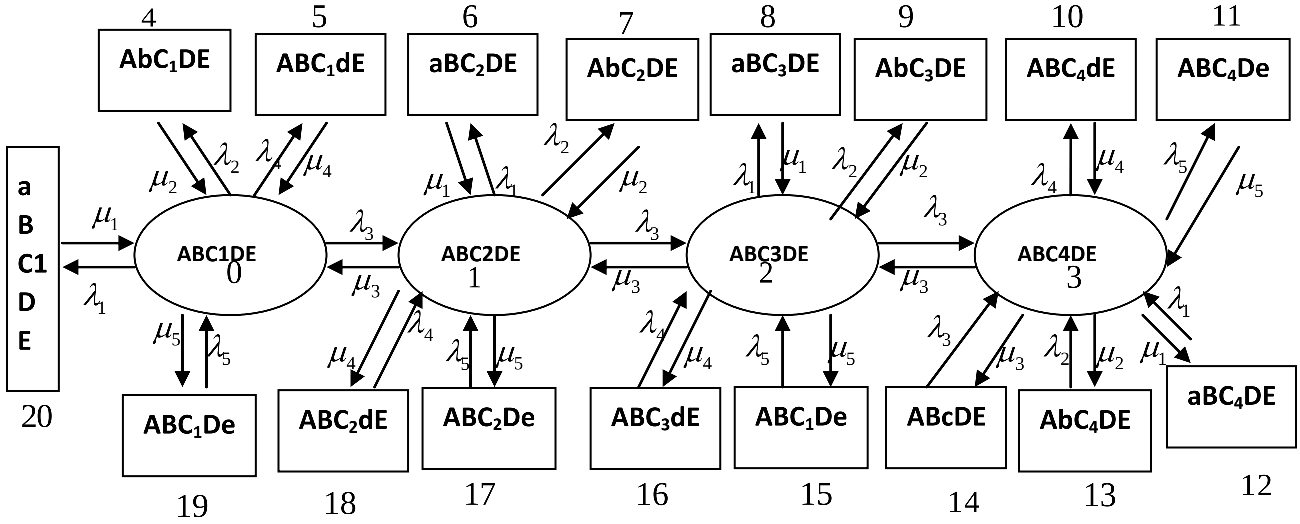

Fig.2. Transition Diagram of the System

3.5 Simulation Modeling

The following differential equations associated with the transition diagram (fig.3.2) are formed:

$$\frac{dp_0(t)}{dt} = -(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)p_0(t)+\mu_1 p_{20}(t)+\mu_2 p_4(t)+\mu_3 p_1(t)+\mu_4 p_5(t)+\mu_5 p_{19}(t) \quad (1)$$

$$\frac{dp_1(t)}{dt} = -(\mu_3+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)p_1(t)+\lambda_3 p_0(t)+\mu_1 p_6(t)+\mu_2 p_7(t)+\mu_3 p_2(t)+\mu_4 p_{18}(t)+\mu_5 p_{17}(t) \quad (2)$$

$$\frac{dp_2(t)}{dt} = -(\mu_3+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)p_2(t)+\lambda_3 p_1(t)+\mu_1 p_8(t)+\mu_2 p_9(t)+\mu_3 p_3(t)+\mu_5 p_{15}(t)+\mu_4 p_{16}(t) \quad (3)$$

$$\frac{dp_3(t)}{dt} = -(\mu_3+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)p_3(t)+\lambda_3 p_2(t)+\mu_1 p_{12}(t)+\mu_2 p_{13}(t)+\mu_3 p_{14}(t)+\mu_4 p_{10}(t)+\mu_5 p_{11}(t) \quad (4)$$

$$\frac{dp_4(t)}{dt} = -\mu_1 p_2(t)+\lambda_2 p_0(t) \quad (5)$$

$$\frac{dp_5(t)}{dt} = -\mu_4 p_5(t)+\lambda_4 p_0(t) \quad (6)$$

$$\frac{dp_6(t)}{dt} = -\mu_1 p_6(t)+\lambda_1 p_1(t) \quad (7)$$

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$$\frac{dp_7(t)}{dt} = -\mu_2 p_7(t) + \lambda_2 p_1(t) \tag{8}$$

$$\frac{dp_8(t)}{dt} = -\mu_1 p_8(t) + \lambda_1 p_2(t) \tag{9}$$

$$\frac{dp_9(t)}{dt} = -\mu_2 p_9(t) + \lambda_2 p_2(t) \tag{10}$$

$$\frac{dp_{10}(t)}{dt} = -\mu_4 p_{10}(t) + \lambda_4 p_3(t) \tag{11}$$

$$\frac{dp_{11}(t)}{dt} = -\mu_5 p_{11}(t) + \lambda_5 p_3(t) \tag{12}$$

$$\frac{dp_{12}(t)}{dt} = -\mu_1 p_{12}(t) + \lambda_1 p_3(t) \tag{13}$$

$$\frac{dp_{13}(t)}{dt} = -\mu_2 p_{13}(t) + \lambda_2 p_3(t) \tag{14}$$

$$\frac{dp_{14}(t)}{dt} = -\mu_3 p_{14}(t) + \lambda_3 p_3(t) \tag{15}$$

$$\frac{dp_{15}(t)}{dt} = -\mu_5 p_{15}(t) + \lambda_5 p_2(t) \tag{16}$$

$$\frac{dp_{16}(t)}{dt} = -\mu_4 p_{16}(t) + \lambda_4 p_2(t) \tag{17}$$

$$\frac{dp_{17}(t)}{dt} = -\mu_5 p_{17}(t) + \lambda_5 p_1(t) \tag{18}$$

$$\frac{dp_{18}(t)}{dt} = -\mu_4 p_{18}(t) + \lambda_4 p_1(t) \tag{19}$$

$$\frac{dp_{19}(t)}{dt} = -\mu_5 p_{19}(t) + \lambda_5 p_0(t) \tag{20}$$

$$\frac{dp_{20}(t)}{dt} = -\mu_1 p_{20}(t) + \lambda_1 p_0(t) \tag{21}$$

Setting $\frac{dp_i(t)}{dt} = 0$ as $t \to \infty$ in equations 3.1 – 3.21 and solve recursively we obtained the following steady state probabilities:

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$$\therefore p_1 = \frac{\lambda_3}{\mu_3} p_o \tag{22}$$

From 3.23

$$\lambda_3 p_0(t) + \mu_1 p_6(t) + \mu_2 p_7(t) + \mu_3 p_2(t) + \mu_4 p_{18}(t) + \mu_5 p_{17}(t) = (\mu_3 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) p_1(t)$$
$$\Rightarrow \lambda_3 p_0 + \mu_3 p_2 = (\mu_3 + \lambda_3) p_1$$

Substituting for

$p_1 = \frac{\lambda_3}{\mu_3} p_0$ we've that

$$p_2 = \frac{\lambda_3^2}{\mu_3^2} p_0$$

$$\therefore p_2 = \left(\frac{\lambda_3}{\mu_3}\right)^2 p_0 \tag{23}$$

From equation 3.24

$$\lambda_3 p_1(t) + \mu_1 p_8(t) + \mu_2 p_9(t) + \mu_3 p_3(t) + \mu_5 p_{15}(t) + \mu_4 p_{16}(t) = (\mu_3 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) p_2(t)$$
$$\Rightarrow \lambda_3 p_1 + \mu_3 p_3 = (\mu_3 + \lambda_3) p_2$$

Substituting for $p_1 and\ p_2$, we have that,

$$p_3 = \left(\frac{\lambda_3}{\mu_3}\right)^3 p_0 \tag{24}$$

$$p_4 = \frac{\lambda_2}{\mu_2} p_0 \tag{25}$$

$$p_5 = \frac{\lambda_4}{\mu_4} p_0 \tag{26}$$

$$p_6 = \frac{\lambda_1}{\mu_1} p_1 = \frac{\lambda_1}{\mu_1} \times \frac{\lambda_3}{\mu_3} p_0 = \frac{\lambda_1 \lambda_3}{\mu_1 \mu_3} p_0 \tag{27}$$

$$p_7 = \frac{\lambda_2}{\mu_2} p_1 = \frac{\lambda_2}{\mu_2} \times \frac{\lambda_3}{\mu_3} p_0 = \frac{\lambda_2 \lambda_3}{\mu_2 \mu_3} p_0 \tag{28}$$

$$p_8 = \frac{\lambda_1}{\mu_1} p_2 = \frac{\lambda_1}{\mu_1} \left(\frac{\lambda_3}{\mu_3}\right)^2 p_0 \tag{29}$$

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$$p_9 = \frac{\lambda_2}{\mu_2} p_2 = \frac{\lambda_2}{\mu_2}\left(\frac{\lambda_3}{\mu_3}\right)^2 p_0 \tag{30}$$

$$p_{10} = \frac{\lambda_4}{\mu_4} p_3 = \frac{\lambda_4}{\mu_4}\left(\frac{\lambda_3}{\mu_3}\right)^3 p_0 \tag{31}$$

$$p_{11} = \frac{\lambda_5}{\mu_5} p_3 = \frac{\lambda_5}{\mu_5}\left(\frac{\lambda_3}{\mu_3}\right)^3 p_0 \tag{32}$$

$$p_{12} = \frac{\lambda_1}{\mu_1} p_3 = \frac{\lambda_1}{\mu_1}\left(\frac{\lambda_3}{\mu_3}\right)^3 p_0 \tag{33}$$

$$p_{13} = \frac{\lambda_2}{\mu_2} p_3 = \frac{\lambda_2}{\mu_2}\left(\frac{\lambda_3}{\mu_3}\right)^3 p_0 \tag{34}$$

$$p_{14} = \frac{\lambda_3}{\mu_3} p_3 = \left(\frac{\lambda_3}{\mu_3}\right)^4 p_0 \tag{35}$$

$$p_{15} = \frac{\lambda_5}{\mu_5} p_2 = \frac{\lambda_5}{\mu_5}\left(\frac{\lambda_3}{\mu_3}\right)^2 p_0 \tag{36}$$

$$p_{16} = \frac{\lambda_4}{\mu_4} p_2 = \frac{\lambda_4}{\mu_4}\left(\frac{\lambda_3}{\mu_3}\right)^2 p_0 \tag{37}$$

$$p_{17} = \frac{\lambda_5}{\mu_5} p_1 = \frac{\lambda_5}{\mu_5}\left(\frac{\lambda_3}{\mu_3}\right) p_0 \tag{38}$$

$$p_{18} = \frac{\lambda_4}{\mu_4} p_1 = \frac{\lambda_4}{\mu_4}\left(\frac{\lambda_3}{\mu_3}\right) p_0 \tag{39}$$

$$p_{19} = \frac{\lambda_5}{\mu_5} p_0 \tag{40}$$

$$p_{20} = \frac{\lambda_1}{\mu_1} p_0 \tag{41}$$

Probability of full working states p_0 is determined using condition normalizing below:

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$$\sum_{i=0}^{20} p_i = 1 \tag{42}$$

i.e

$$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 + p_{10} + \ldots + p_{20} = 1 \tag{43}$$

$$\therefore p_0 = \frac{1}{1+\left(X_1 + X_2 + X_3 + X_4 + X_5\right)\left(1+ X_3 + X_3^2 + X_3^3\right)} \tag{44}$$

6. Availability Analysis

The steady state availability of the system may be obtained as summation of all working states and reduced capacity state probabilities as:

$$A_V = \sum_{i=0}^{3} p_i \tag{45}$$

$$\Rightarrow A_V = p_0 + p_1 + p_2 + p_3 \tag{46}$$

$$= p_0 + X_3 p_0 + X_3^2 p_0 + X_3^3 p_0$$
$$= p_0\left(1+ X_3 + X_3^2 + X_3^3\right)$$

$$\therefore A_V = \frac{1+ X_3 + X_3^2 + X_3^3}{1+\left(X_1 + X_2 + X_3 + X_4 + X_5\right)\left(1+ X_3 + X_3^2 + X_3^3\right)} \tag{47}$$

7. Busy Period Analysis

Busy period is the summation of probabilities of failure and reduced states where the repair man is busy repairing the failed units/subsystems,

$$B_P = \sum_{K=1}^{20} p_K, \tag{48}$$

$$B_P = p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 + p_{10} + \ldots + p_{20} \tag{49}$$

$$\therefore B_P = \frac{\left(X_1 + X_2 + X_3 + X_4 + X_5\right)\left(1+ X_3 + X_3^2 + X_3^3\right)}{1+\left(X_1 + X_2 + X_3 + X_4 + X_5\right)\left(1+ X_3 + X_3^2 + X_3^3\right)} \tag{50}$$

8. Profit Analysis

$P_F =$ Total revenue generated – Total cost incurred due to repair

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$$\Rightarrow P_F = C_0 A_V - C_1 B_P \text{, where} \tag{51}$$

$C_0 = £1000$ and $C_1 = £100$

$$P_F = C_0 \left[\frac{1 + X_3 + X_3^2 + X_3^3}{1 + (X_1 + X_2 + X_3 + X_4 + X_5)(1 + X_3 + X_3^2 + X_3^3)} \right] - C_1 \left[\frac{(X_1 + X_2 + X_3 + X_4 + X_5)(1 + X_3 + X_3^2 + X_3^3)}{1 + (X_1 + X_2 + X_3 + X_4 + X_5)(1 + X_3 + X_3^2 + X_3^3)} \right] \tag{52}$$

RESULTS AND DISCUSSION

Tables 1-5 and Figure 3. through Figure 12. reveal the effect of failure and repair rates of subsystems A, B, C, D and E on profit of the system. It is observed that for some known constant values of failure / repair rates of the subsystems, as failure rate of subsystem increases, the subsystem profit decreases. Similarly as repair rate of the subsystem increases, the subsystem profit also increases.

Table 1. Profit Matrix of Subsystem A

λ_1 / μ_1	0.1000	0.1850	0.2700	0.3550	0.4400
0.0500	377.73	432.90	456.71	469.98	478.44
0.0680	341.89	408.05	437.88	454.85	465.81
0.0860	311.05	385.42	420.28	440.51	453.72
0.1040	284.23	364.71	403.80	426.89	442.14
0.1220	260.70	345.70	388.33	413.94	431.03

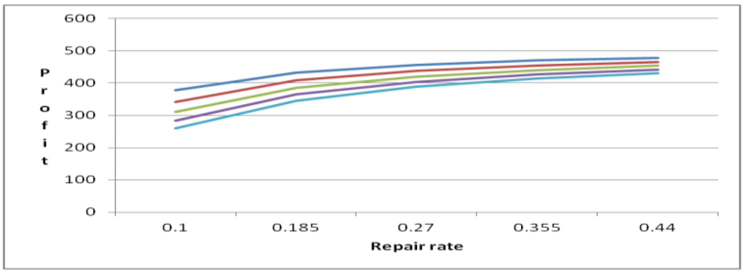

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Fig.3. Effect μ_1 on Profit

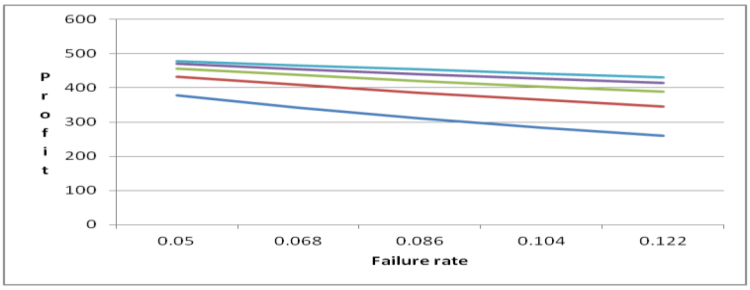

Fig. 4. Effect of λ_1 on Profit

Table 2. Profit Matrix of Subsystem B

λ_2 / μ_2	0.1000	0.2000	0.3000	0.4000	0.5000
0.0750	343.20	424.33	458.42	477.17	489.04
0.1000	301.75	394.18	435.23	458.42	473.31
0.1250	267.40	367.30	413.89	440.84	458.42
0.1500	238.45	343.20	394.18	424.34	444.27
0.1750	213.74	321.46	375.93	408.81	430.82

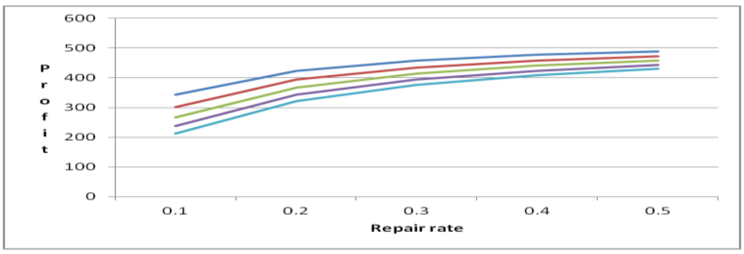

Fig. 5. Effect of μ_2 on profit

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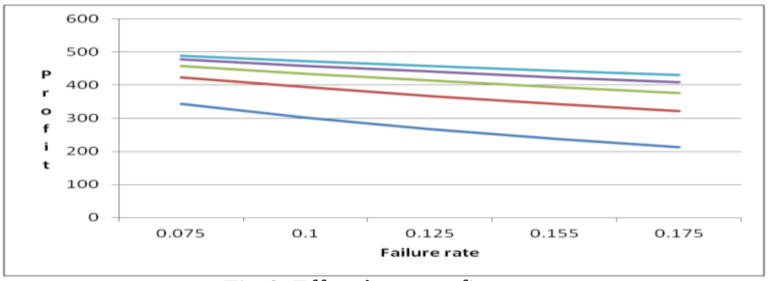

Fig.6. Effect λ_2 on profit

Table 3. Profit Matrix of Subsystem C

λ_3 / μ_3	0.1250	0.2000	0,2750	0.3500	0.4250
0.0400	383.34	389.81	392.45	393.90	394.83
0.0575	373.95	385.21	389.36	391.55	392.93
0.0750	362.11	379.93	386.03	389.10	390.97
0.9250	347.96	373.76	382.36	386.49	388.93
0.1100	332.10	366.62	378.26	383.68	386.79

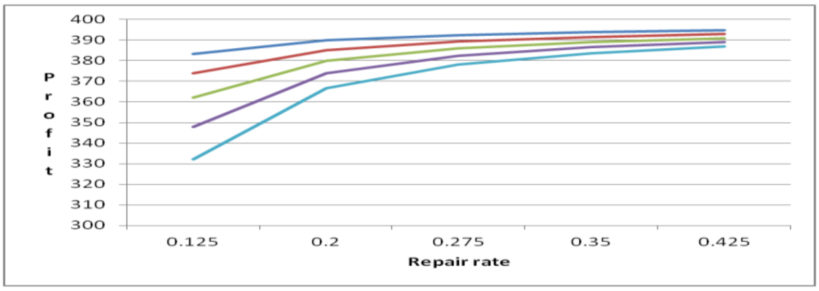

Fig.7. Effect of μ_3 on profit

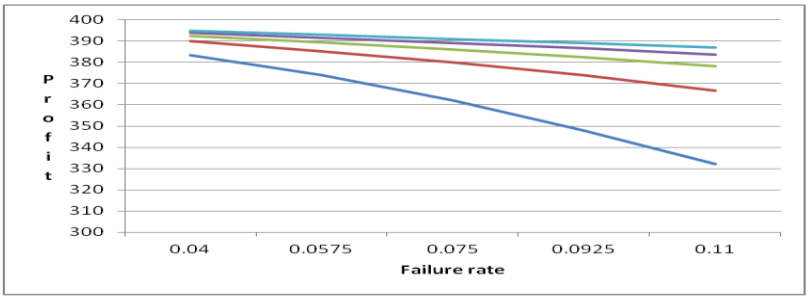

Fig.8. Effect of λ_3 on profit

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Table 4. Profit Matrix of Subsystem D

λ_4 / μ_4	0.1550	0.2300	0.3050	0.3800	0.4550
0.0500	522.47	562.51	584.90	599.20	609.13
0.0650	489.68	537.10	564.25	581.83	594.15
0.0800	460.17	513.57	544.80	565.30	579.79
0.0950	433.47	491.71	526.47	549.56	566.01
0.1100	409.21	471.36	509.14	534.54	552.78

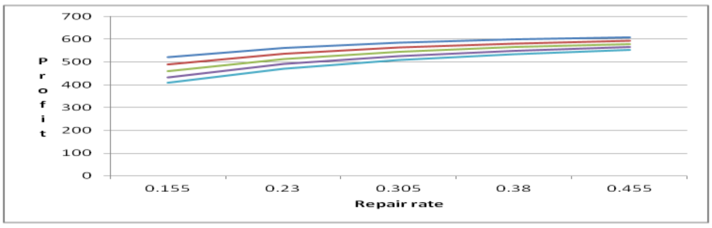

Fig.9. Effect of μ_4 on profit

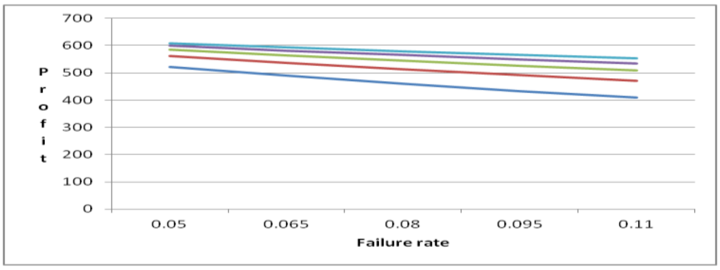

Fig.10. Effect of λ_4 on profit

Table 5. Profit Matrix of Subsystem E

λ_5 / μ_5	0.1000	0.1950	0.2900	0.3850	0.4800
0.0550	343.58	398.30	420.44	432.42	439.92
0.0660	324.45	385.69	411.12	425.03	433.82
0.0770	306.90	373.70	402.12	417.86	427.86
0.0880	290.74	362.29	393.44	410.88	422.02
0.0990	275.82	351.41	385.05	404.08	416.32

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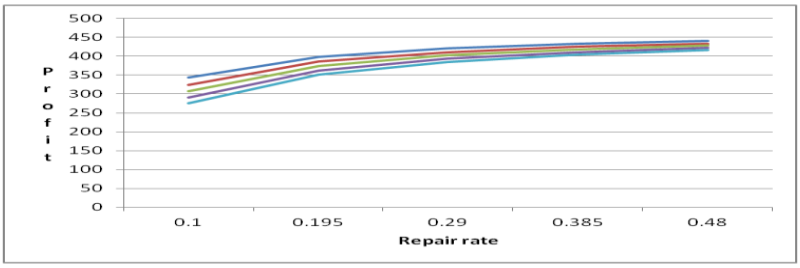

Fig.11. Effect of μ_5 on profit

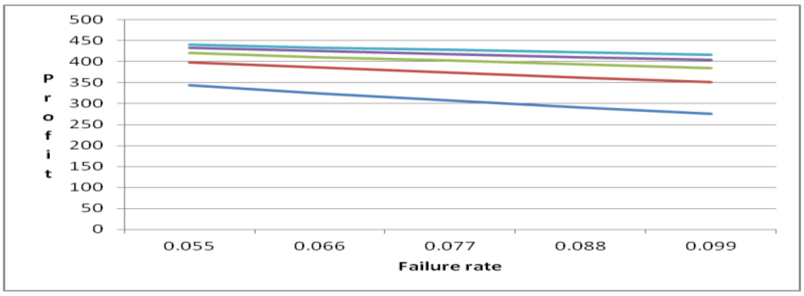

Fig.12. Effect of λ_5 on profit

Table 6. Optimum values of Failure/Repair rates of Subsystems

S/N	Subsystem	Failure rate λ_i	Repair rate μ_i	Maximum Profit Level	Maximum Availability
1	A	0.0500	0.4400	478.44	54%
2	B	0.0750	0.5000	489.04	54%
3	C	0.0400	0.4250	394.83	45%
4	D	0.0500	0.4550	609.13	65%
5	E	0.0550	0.4800	439.92	43%

Table 6. helps in determining the subsystem with maximum profit and availability. It is observed that subsystem D is having maximum profit and availability. Shown in the Table 4.16 are the optimum values of failure and repair rates for maximum profit and availability for each subsystem.

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CONCLUSION

Explicit expression for the reliability model is developed and used for the evaluation of performance of different subsystems of the series-parallel system in this study. Using the model, tables 1- 5 are constructed to show the relationship between failure and repair rates on system performance, profit. The profit/Availability decreases as the failure rate increases. Similarly as profit/Availability increases so also the repair rates. The model will assist maintenance engineers and managers for proper maintenance utilization. The results of this study will be beneficial to the plant management for the reliability, and profit analysis of a complex standby redundant system and hence to decide about the maintenance priorities of various subsystems of any similar system.

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